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By- Balabanian, Norman

RESISTIVE NETWORKS.

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Norman Balabanian

Electrical Engineering Department

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Generally speaking, engineers and scientists perform two different kinds of work: analysis, or the examination of a whole structure in order to understand its parts and their relationships; and synthesis or design, or the combination of separate parts into a whole whose characteristics have been prescribed beforehand.

Many of the problems which an engineer is called upon to solve involve finding either the voltage across, the current, or the power absorbed by an electrical device when the device is connected in a network of other devices and electrical sources of energy. This is a problem of analysis. Conversely, an engineer may be asked to specify the characteristics of one electrical device in a network of other devices, or the amount of power a given source is required to deliver, in order that the voltage, current, or power absorbed somewhere in the network be equal to a specified value. This is a problem of synthesis or design.

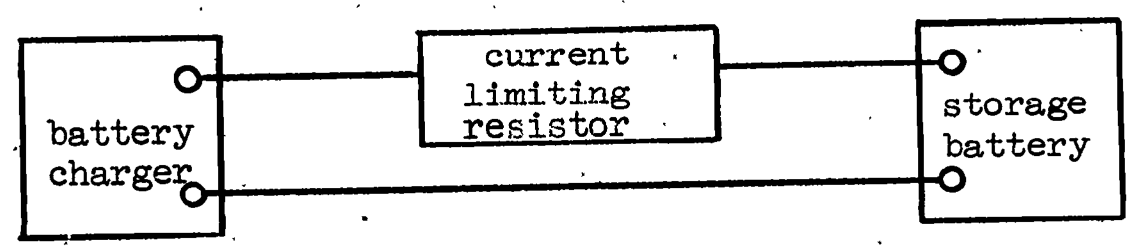
Up to this point, we have introduced three hypothetical devices (models) and several laws. We have considered an ideal resistor, an ideal voltage source, and an ideal current source. (We will often abbreviate these as v-source and i-source.) We have also introduced Kirchhoff's and Ohm's laws which describe the interrelations of voltage and current in a network containing these three models (ideal devices). Now, if the behavior of real, physical devices in a network can be adequately represented by combinations of these models, the result will be a network of resistors, v-sources, and i-sources to which we can apply Kirchhoff's and Ohm's laws.

In this unit we shall discuss methods which will lead to the successful solution of problems of analysis and design involving networks containing only resistors, voltage sources, and current sources. Specifically, by the end of this unit you should be able to do the following things:

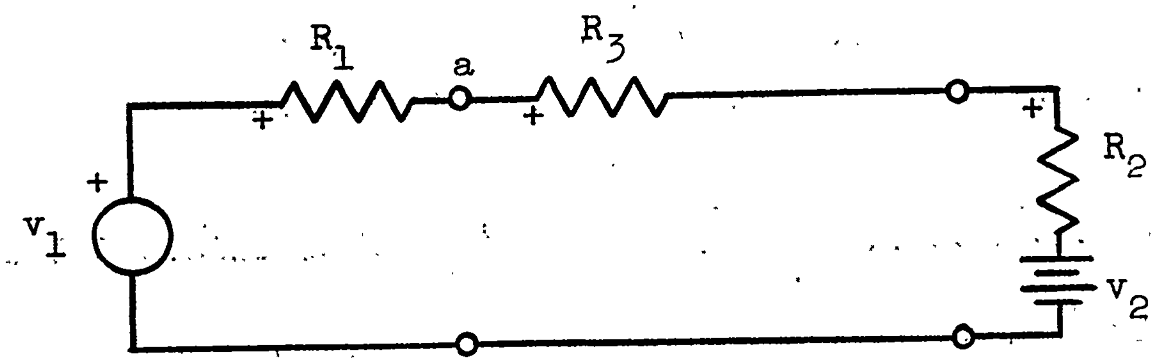
1. Find the voltages and/or currents and/or power dissipated in one or more branches of a circuit, or the power delivered by one or more sources, when given a network of moderate complexity (up to 3 closed paths or 10 branches) containing resistors, v-sources and i-sources whose values are specified (Analysis).

2. Find the values of unspecified resistances or sources, when given a network (similar to those above) in which an appropriate number of branch voltages, currents, or power dissipation is specified (<u>Synthesis</u> or <u>Design</u>).

3. Do both of the above by any of the following methods:
   a) Reduction of a network to an equivalent network;
   b) Alternate application of Kvl and Kcl and Ohm's law;
   c) Direct application of Kvl, leading to equations into which Ohm's law and Kcl are substituted, resulting in equations involving only current variables (i.e., "loop" equations).
   d) Direct application of Kcl, leading to equations into which Ohm's law and Kvl are substituted, resulting in equations involving only voltage variables (i.e., "node" equations).


battery
charger
current
limiting
resistor
storage
battery

(a)


R1
a
R3
R2
v1
v2
+
+
+
+

(b)

Figure 1

Section 1

Let's start the discussion with the situation portrayed in Fig. 1. A battery charger is to charge a storage battery through a physical resistor whose function is to control the amount of current.

The first task is to replace the physical circuit by a model. We have seen that the physical battery can be replaced by an ideal voltage source in series with a resistor. Let us assume that the battery charger can also be represented by a similar combination, and that the physical resistor is adequately represented by an ideal resistor. The resulting circuit is also shown in Fig. 1.

Letting v_{R_1}, v_{R_2} and v_{R_3} be the voltages across the three resistors, with the references shown, write Kvl for this circuit in terms of all the voltages starting at the point labeled a.

6

Answer:

$$v_{R_3} + v_{R_2} + v_2 - v_1 + v_{R_1} = 0$$

(or the same thing multiplied by -1 if you went counter-clockwise.)

Next, note how the current in the components are related to each other; they are ____________.

Branches which are connected end-to-end so that they carry the same current are said to be in series. A network in which the branches are connected as in Fig. 1 is called a ____________ network.

8

Answer:

the same (or equal, or identical)

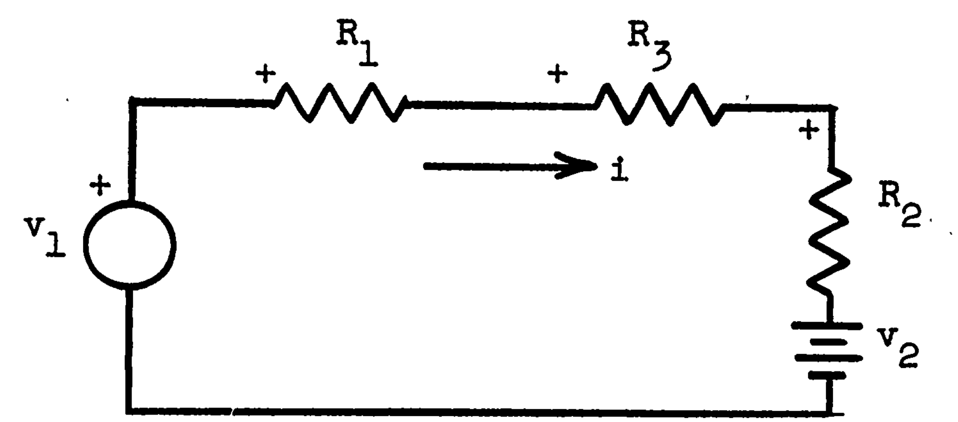
series

9

Redraw Fig. 1 and choose a reference direction for the current. Then apply Ohm's law to substitute current functions for v_{R_1}, v_{R_2} and v_{R_3} in the equation.

10

Answer:



$R_3 i + R_2 i + v_2 - v_1 + R_1 i = 0$

(You may have chosen the opposite reference for current in which case all the terms containing i would have the opposite sign.)

The terms in this equation can be regrouped and the current factored as follows:

$$(v_1 - v_2) = (R_1 + R_2 + R_3)\ i$$

The form of this expression is the same as that of a $v = Ri$ resistor with a <u>single</u> voltage across it.

A number of observations can be made by comparing the above two equations:

1. When there is more than one voltage source in a series network, even though there are intervening resistors, the v-sources can be replaced by a single equivalent voltage source. This <u>equivalent</u> voltage is equal to the algebraic sum of all the source voltages.
2. A series connection of resistors can be replaced by ______________ ____ ____.

12

Answer:

A series connection of resistors can be replaced by <u>a single equivalent resistor whose value equals the sum of the series resistances</u>.

13

Four resistors whose resistances are 800 ohms, 1.5 kilohms, 4 kilohms and 700 ohms are connected in series. Draw a diagram showing this connection and find the equivalent resistance R. (Does the order in which the resistors are connected influence the result?)

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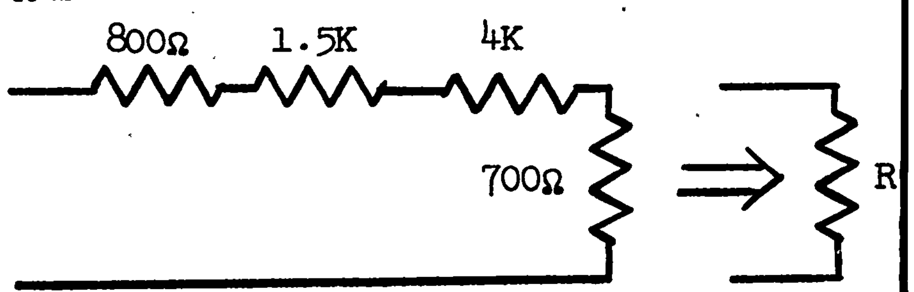
Answer:



$R = .8 + 1.5 + 4 + .7 = \underline{7 \text{ kilohms}}$.

(<u>Connection order is immaterial</u>.)

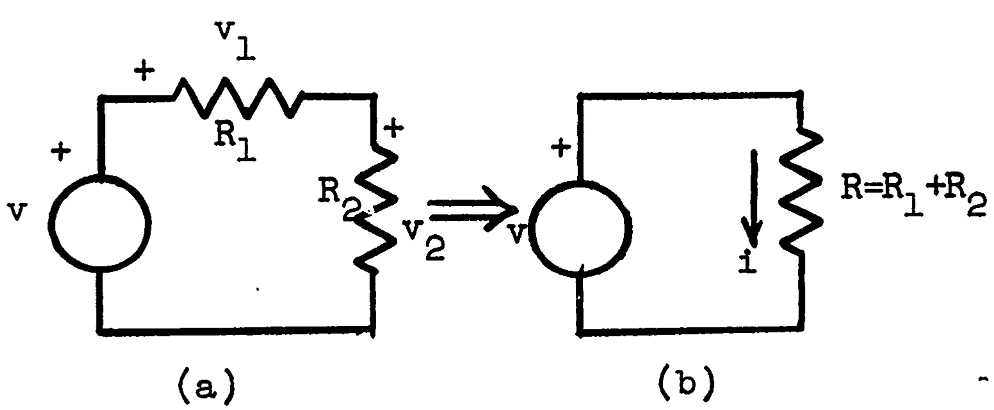


Figure 2

Whenever resistors are connected in series they can be combined into a single equivalent resistance, as already discussed. If there is a voltage across the series combination, the resulting current can be found by Ohm's law. However, suppose it is required to find the voltage across just one of the series connected resistors. For example, suppose it is desired to find the voltage across R_2 in Fig. 2. If the two resistors are combined into an equivalent resistor, the identity of R_2 is lost. Nevertheless, once the current <u>i</u> is determined from Fig. 2(b), the voltage v_2 can be calculated from Fig. 2(a) to be

$$v_2 = R_2 i = R_2 \frac{v}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} v$$

In the same way, the voltage v_1 across R_1 is

$$v_1 =$$

16

Answer:

$$v_1 = \frac{R_1}{R_1 + R_2} v$$

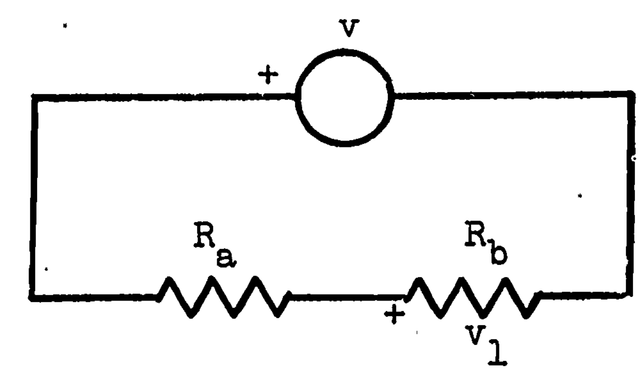


Figure 3

Thus, the voltage v divides itself between the two resistors. This leads to the name voltage divider for the structure we have been discussing (i.e., resistors in series, with a voltage across the combination.) This structure arises often in networks, and remembering the relationships for the voltages, as you just derived them, helps in solving network problems.

An easy way to remember the voltage divider formula is to think of a proportionality:

> "The voltage across one resistor is to the voltage across the series combination what the value of that resistance is to the total resistance."

Using the voltage divider formula, write the expression for v_1 in Fig. 3.

18

Answer:

$$\frac{v_1}{v} = \frac{R_b}{R_b + R_a}$$

or

$$v_1 = \frac{R_b}{R_b + R_a} v$$

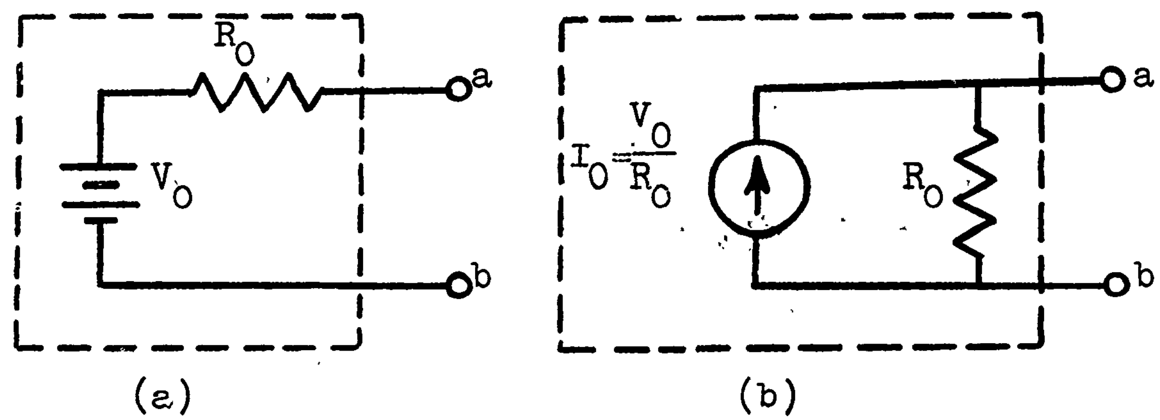


Figure 4

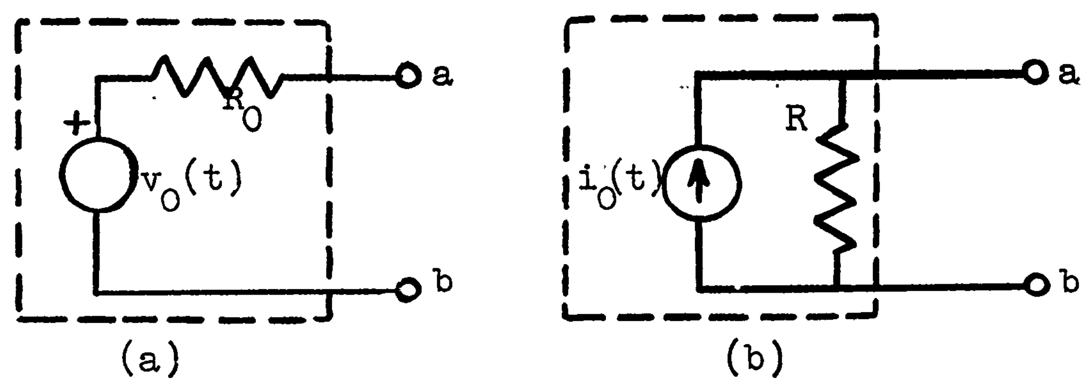


Figure 5

Let's digress for a moment.

In a previous program (Ohm's Law and Sources) it was shown that a physical battery can be represented adequately by either of two equivalent networks as shown in Fig. 4. From the outside, these two networks of hypothetical devices act the same -- that is, they are said to be equivalent at the terminals a-b. On the inside, we have assumed that V_0 and I_0 are constant and that $V_0 = R_0 I_0$.

We now inquire whether this equivalence holds more generally. That is, looking at Fig. 5 when v_0 and i_0 are not necessarily constant but may vary with time, i.e., $v_0(t)$ and $i_0(t)$, can we find a relationship which includes them and R_0 and R such that these two networks are equivalent at the terminals?

Write a general statement defining what is means by, "Two networks are equivalent at a pair of terminals."

____

____

____.

20

Answer:

Your answer should imply two things: (1) that the v-i relationship at the terminals of the two networks are the same, and (2) this v-i relationship holds <u>regardless of the actual values</u> of <u>v</u> and <u>i</u>. For example, you might have said:

> "Two networks are equivalent at a pair of terminals if their v-i relations are <u>always identical</u>."

Review your own statement and correct or improve it as needed.

By analogy with the case of the battery, you might suspect that $R = R_O$ and $i_O(t) = v_O(t)/R_O$. If we assume that this is the case, and we determine the voltage-current relationship for each of the two networks and find that they are the same, we will have demonstrated that the two networks are equivalent.

Assume a current i and voltage v at the terminals of each of the networks in Fig. 5 and write the two v-i relationships using the basic laws (Kvl, Kcl or Ohm's law). Then insert the suspected relationships: $R = R_O$ and $i_O(t) = v_O(t)/R_O$. What is your conclusions; are the two networks equivalent?

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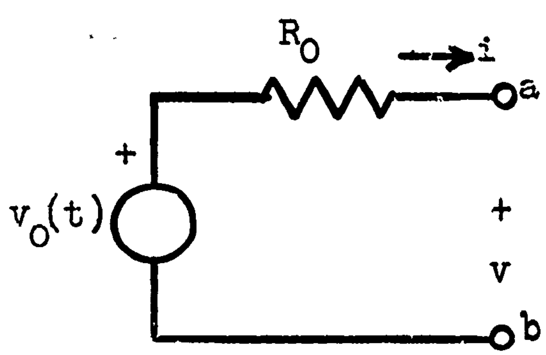
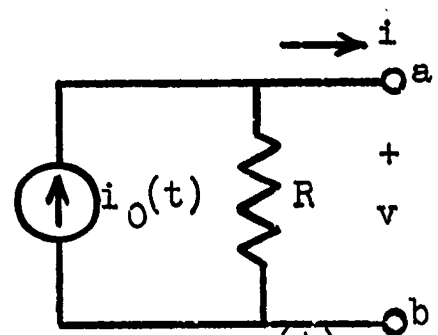
Answer:



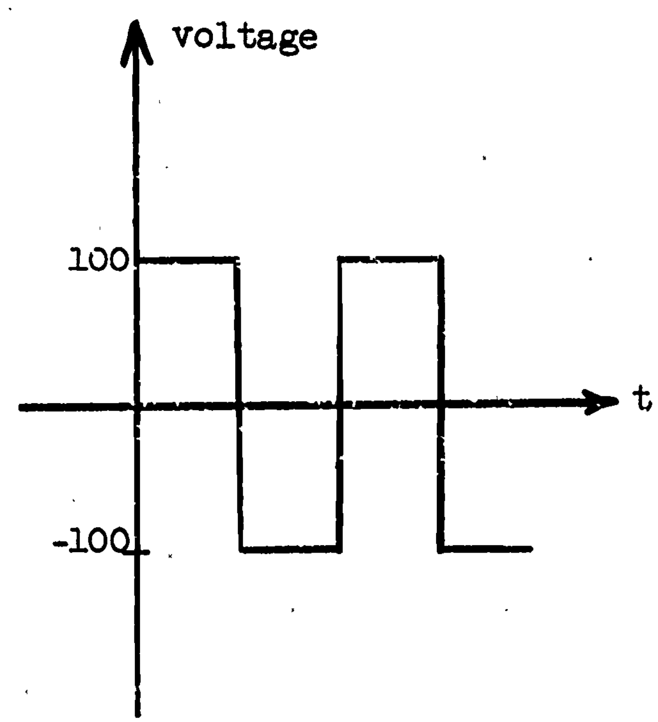
$v = v_O(t) - R_O i$
by Kvl and Ohm's law



$i = i_O - \frac{v}{R} = \frac{v_O(t)}{R_O} - \frac{v}{R_O}$

or $v = v_O(t) - R_O i$
by Kcl and Ohm's law

The two v-i relationships are <u>identical</u> and the networks are <u>equivalent</u>.

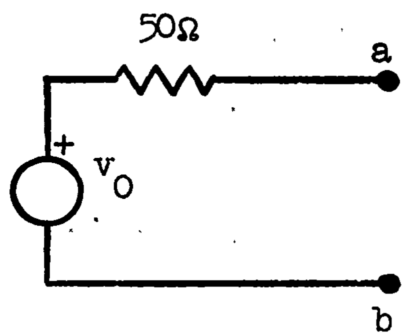


By measurements made at the terminals of a physical laboratory source, it is found that the internal resistance is 50 ohms and that, with a 50 ohm resistor connected externally to the terminals, the voltage is a square wave with a peak-to-peak voltage of 200 volts, as shown in the diagram.

Draw both a v-source and an i-source equivalent of the laboratory source. Label the parts and specify appropriate values either numerically or by a graph.

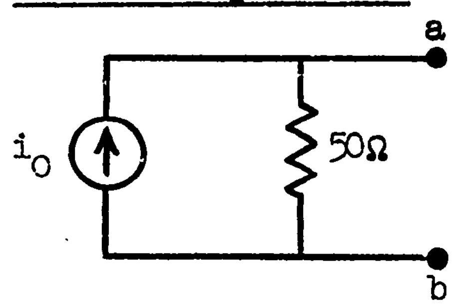
Answer:

v-source equivalent



v_o is a square wave with with a peak-to-peak voltage of 400 volts. (This can be obtained from the voltage divider formula.)

i-source equivalent



i_o is a square wave with a peak-to-peak current of 8 amps.

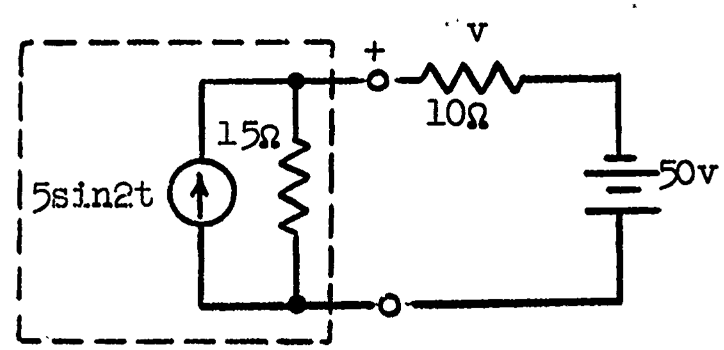


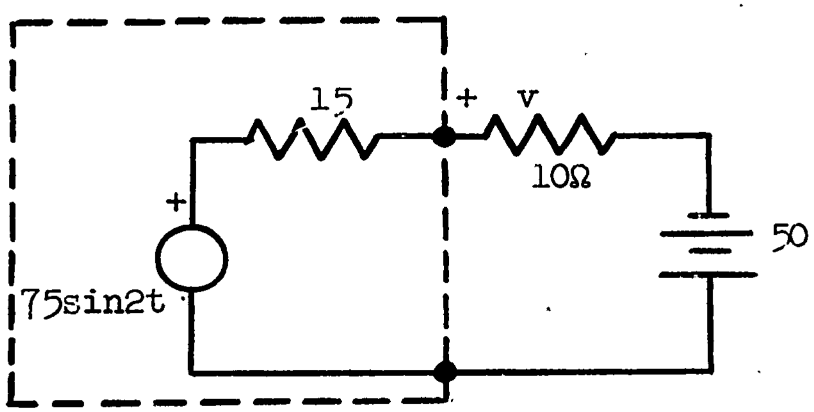
Figure 6

Let us now use this v- and i-source equivalence to permit simplification of a network so that it may be easily solved.

It is desired to find the voltage v across the external 10 ohm resistor in Fig. 6. We might be tempted to use the voltage-divider formula but, in its present form, this is not a simple series network.

Redraw the network using the equivalence of sources to get a series network. Then use the voltage divider formula to find an expression for v. When considering what happens externally, the two networks internal to the dashed lines are equivalent.

Answer:



$$v = \frac{10}{25}(75 \sin 2t + 50)$$

$$= (30 \sin 2t + 20) \text{ volts}$$

Figure 6(a)

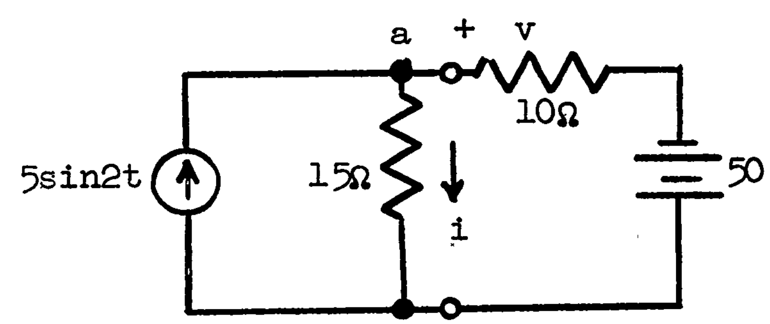


Figure 6 (repeated)

Suppose that in the original network of Fig. 6 it is desired to determine the current in the internal 15 ohm resistor. One might be tempted to find it from the equivalent network in Fig. 6(a) where the current in the 15 ohm resistor is the same as that in the 10 ohm resistor. This external current can be easily found.

Another person might return to the original network (repeated on page 26) and note that, with v now known, the current in the 10 ohm resistor is known; applying Kcl to the node labeled a will then lead to i.

Carry out both of these suggestions. Compare the answers and comment on their appropriateness in answering the question.

current in 15 ohm resistor in original network, Fig. 6, is ____________.

current in 15 ohm resistor in "equivalent" network, Fig. 6(a), is ____________.

Answer: In the original network, $i = 2 \sin 2t - 2$. In the network of Fig. 6(a), $i = 3 \sin 2t + 2$. Clearly, these currents are not identical! Your comments should have noted that, although these two networks may be equivalent at the terminals (externally), they are not internally identical. The original question concerned current in the 15 ohm resistance of Fig. 6. Only the first of the two expressions above, answers the question that was asked.

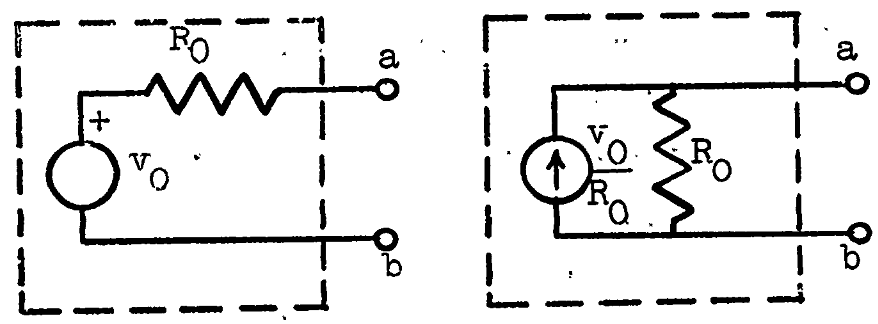


Figure 7

This example serves as a warning that when a portion of a network (a sub-network) is replaced by an equivalent, careful observance of the meaning of equivalence must be maintained. Thus, in Fig. 7, when we say these two networks are equivalent, we mean that, so far as the terminals a, b are concerned, the relationship between the voltage and current is the same for the two. We say nothing about conditions within the dashed boxes, internal to the two networks.

In our example, the preceding remarks mean that when we replace the subnetwork inside one of the dashed boxes by the other one, we can solve for only those voltages and currents <u>outside</u> these boxes. Thus, the current in the 15 ohm resistor in the one case cannot be determined by using the second network.

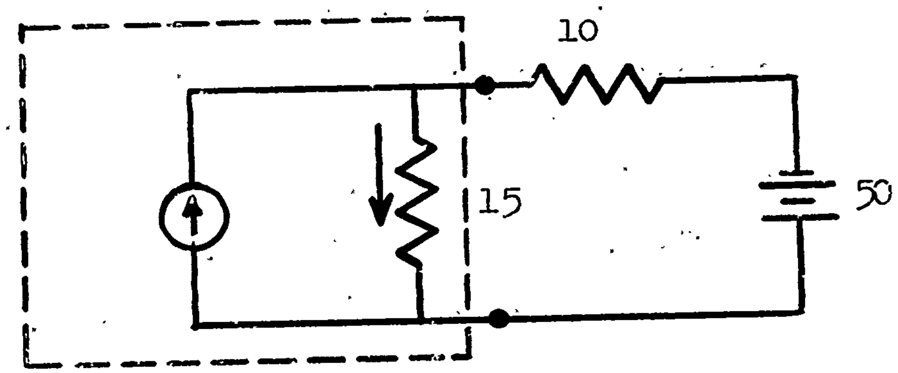


Figure 6 (repeated)

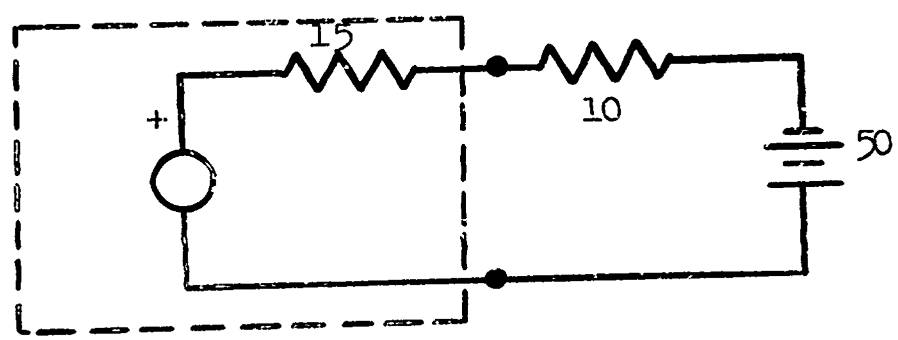


Figure 6(a)(repeated)

Let us return again to the original network of Fig. 6, which is repeated on page 30. Suppose that our interest is in determining the current in the 15 ohm resistor.

Earlier (when we were interested in the voltage across the 10 ohm resistor), we replaced the current source and 15 ohm resistance by an equivalent network which led to a simple series network.

Let us now, instead, replace the other part of the network, the 50 volt battery and 10 ohm resistance, with an appropriate equivalent. Draw a diagram of the resulting network and label it appropriately.

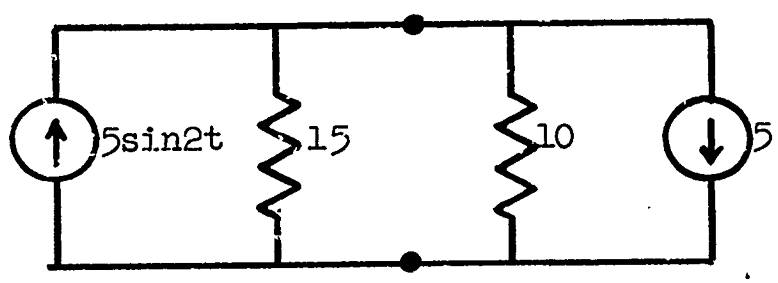


Figure 6(b)

Figure 6(b)

On the following pages, we will discuss the solution of this type of problem.

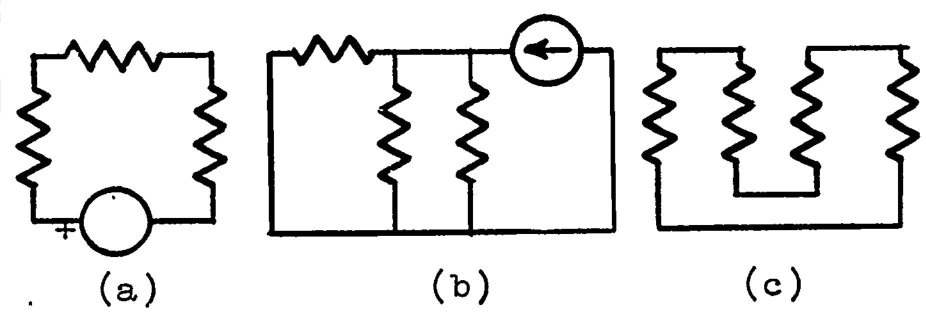


Figure 8

The structure of this network (Fig. 6(b)) is such that the voltage across each of the branches is the same. Branches which are connected so that their voltages are all the same are said to be in parallel and the resulting networks are said to be parallel networks.

State which ones, if any, of the networks in Fig. 8 are in parallel. State how the remaining ones are connected.

Answer:

The branches of (b) are connected in parallel. In (a) and (c) the branches are in series.

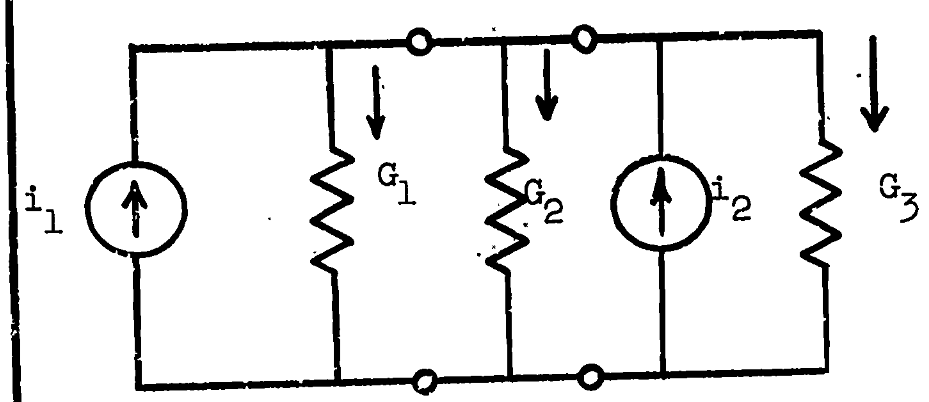


Figure 9

Rather than dealing with a specific numerical example, let us turn to a parallel network having somewhat more generality yet not so complicated that it becomes difficult to handle. Such a network is shown in Fig. 9. The resistors are labeled in terms of their <u>conductances</u> $(G = \frac{1}{R})$.

Let i_{G_1}, i_{G_2} and i_{G_3} be the currents in the three resistors with the references shown. Write Kcl at one of the nodes of this network, starting with the left hand branch.

Answer:

$$-i_1 + i_{G_1} + i_{G_2} - i_2 + i_{G_3} = 0$$

(If you took "toward" as the junction reference and used the upper node, or if you used the lower node with "away from" as the junction reference, you got the opposite signs.)

Redraw the diagram (Fig. 9) choosing a reference for the voltage v. By appropriate substitutions and regrouping of terms, put the previous equation $(-i_1 + i_{G_1} + i_{G_2} - i_2 + i_{G_3} = 0)$ into the form $i = Gv$ and write identifying expressions for i and G.

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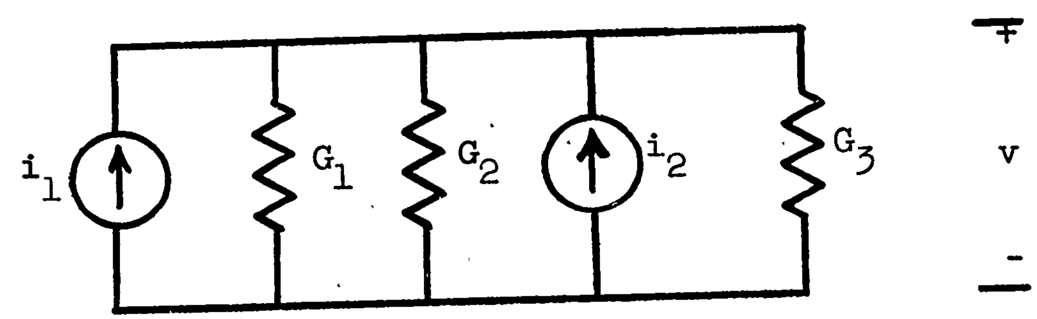
Answer:



$(i_1 + i_2) = (G_1 + G_2 + G_3)\ v$

(If you chose the opposite reference for v, you should have a negative sign on one side of your equation.)

$i = i_1 + i_2$

$G = G_1 + G_2 + G_3$

It is clear that when two or more current sources are connected <u>in parallel</u> they can be replaced by a single <u>equivalent</u> current source equal to the algebraic sum of the source currents. Also, a parallel connection of resistors ____.

Answer:

is equivalent to a single resistor (or can be replaced by a single equivalent resistor) whose conductance is equal to the sum of the individual conductances.

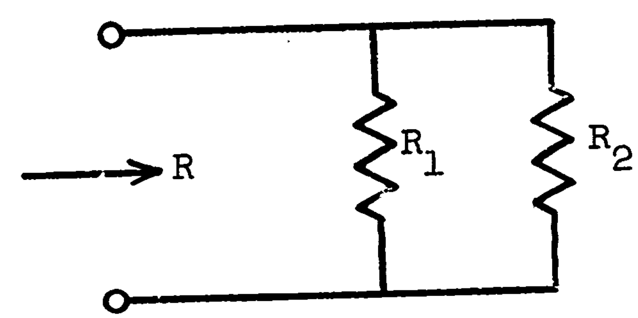


Figure 10

That is, if G_1, G_2, ..., G_n are connected in parallel, a single conductance $G = G_1 + G_2 + \dots G_n$ is equivalent to the parallel combination. Note that this result is stated in terms of conductances whereas resistors are usually specified in terms of their resistances.

Suppose we wish to know the equivalent resistance of two resistors connected in parallel as in Fig. 10. Using the above result, find an expression for the equivalent resistance R in terms of R_1 and R_2.

42

Answer:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

This is obtained by first writing $G = G_1 + G_2$, replacing conductances by one over their reciprocals, and solving for R.

The combination of two resistors in parallel occurs very often and it helps in the solution of problems if this relationship and its properties become well known to you.

One property to know, for example, is the relative value of two resistances and their parallel connection. Let $R_1 = 10$ ohms and $R_2 = 30$ ohms. Determine the equivalent resistance R when these two are in parallel and note its value relative to (greater or less than) that of R_1 and R_2. Then prove that for <u>any</u> two resistance R_1 and R_2 the parallel equivalent resistance has this same relative value.

Answer:

$R = \frac{(10)(30)}{10 + 30} = 7.5$ ohms which is <u>less</u> than both $R_1 = 10$ and $R_2 = 30$. In general, $G = G_1 + G_2$, and since G_1 and G_2 are both positive quantities, $G > G_1$ and $G > G_2$. Putting in the reciprocals leads to $R < R_1$ and $R < R_2$.

Another approach is to take $R - R_1$ and $R - R_2$, with $R = R_1R_2/(R_1 + R_2)$ and to note whether the results are positive or negative. If positive, R must be greater; if negative R must be less. (Try this, if you did not do it this way.)

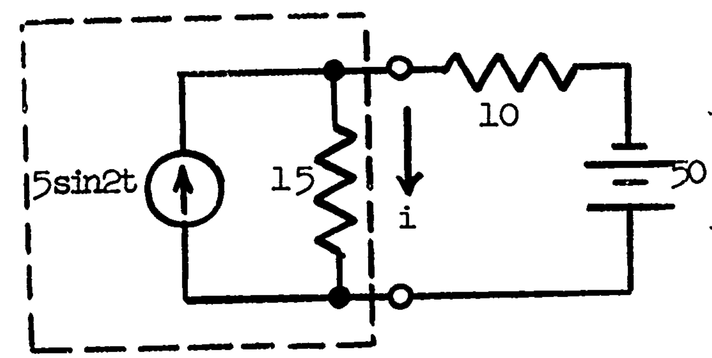
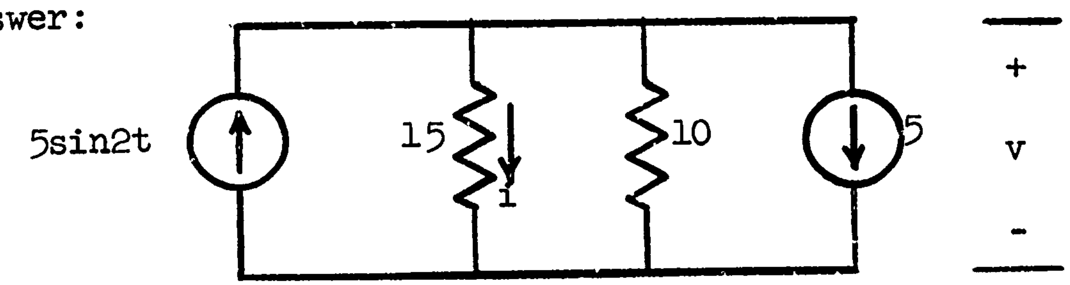


Figure 6 (repeated)

Having introduced a parallel network and discussed some of its general properties, let us return to the example previously under consideration to determine the current i in the 15 ohm resistor. The diagram is repeated on page 44. This network is neither a series network nor a parallel network. It was earlier converted to a series network by replacing the current source in parallel with the 15 ohm resistor by an equivalent consisting of an appropriate voltage source in series with the resistor. Alternatively, the network is converted into a parallel network if the battery in series with the 10 ohm resistor is replaced by an equivalent consisting of an appropriate current source in parallel with the resistor.

Repeat this latter and determine the current i in the 15 ohm resistor. Verify by comparing with the value determined on page 23.

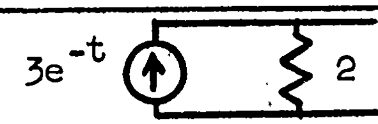
Answer:



i = 2sin2t-2

This is obtained by determining (1) the equivalent current source of the two in parallel: 5 sin 2t - 5; (2) equivalent resistance of the two in parallel: 10(15)/(10 + 15) = 6 ohms; (3) the voltage v across each of the branches: 6 times 5 sin 2t - 5; and, finally, (4) the current i equals v/15.

To summarize:

1. In a series network of resistors the equivalent resistance is ______ ____.
2. In a voltage divider, the voltage across one resistor is __________ ____.
3. For this network 

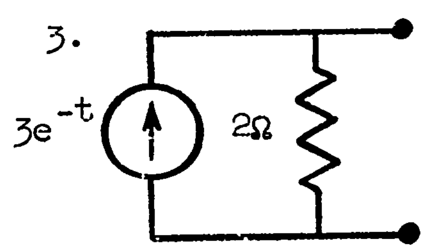
an equivalent network is ______________. (Draw a diagram and label the parts.)
4. Two resistors whose <u>conductances</u> are 1000μ mhos and 4000μ mhos are connected in parallel. The <u>resistance</u> of the combination is R = ______________.

Answer:

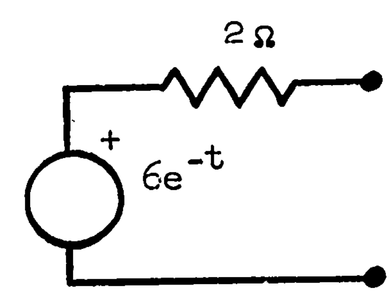
1. the equivalent resistance is <u>equal to the sum of the series resistances</u>.

2. the voltage across one resistor is <u>to the total voltage what that resistance is to the total resistance</u>.

3.



is equivalent to



4. R = <u>200 ohms</u>.

We suggest you take a break at this point, before going on.

<u>Section 2</u>

Looking back over the work of this unit, it appears that solving for an unknown voltage or current either in a series circuit or in a parallel combination is relatively simple. We also saw an example in which it was possible to convert a given network into a series network or a parallel network. This was done by replacing a subnetwork, consisting of a current source in parallel with a resistor, by an appropriate voltage source in series with the resistor. (For ease of reference we shall call these two, shown again in Fig. 11, <u>equivalent sources</u>. Note carefully that the term applies to the combination of source and resistance.) We say that the first is the <u>voltage source equivalent</u> of the second and the second is the <u>current source equivalent</u> of the first. (Repeat these names out loud.)

50

Not all networks can be converted to a simple series or parallel network by equivalent sources. We shall now discuss a procedure which combines the use of equivalent sources with parallel combinations and series combinations of resistances to reduce a relatively complicated network to a simple series or parallel form.

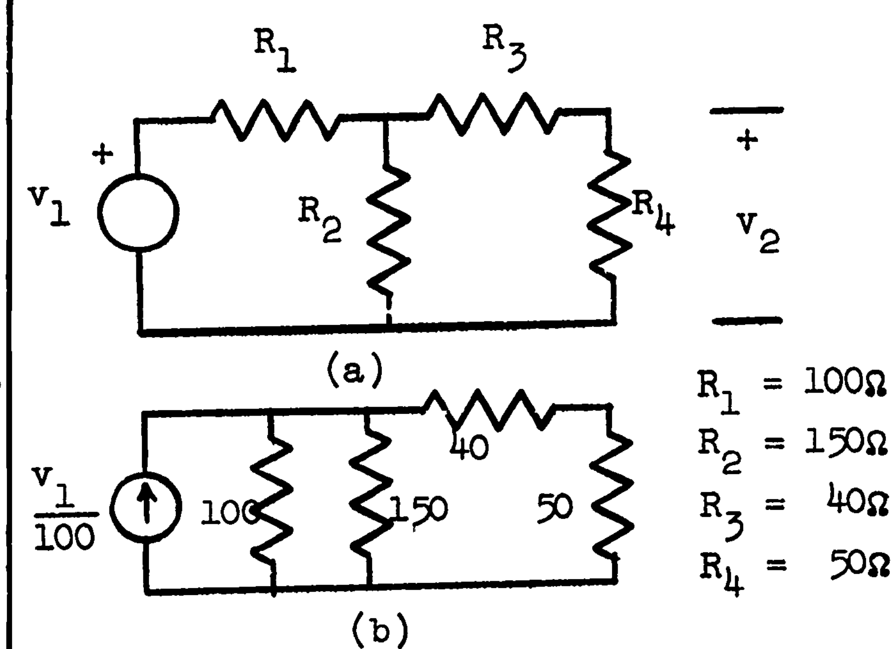


Figure 12

To start the discussion, consider the network shown in Fig. 12(a). It is desired to determine the voltage v_2 across resistor R_4. For convenience, let us initially suppose the resistors have the numerical values shown, but focus your attention on the <u>process</u> being carried out rather than the numerical results.

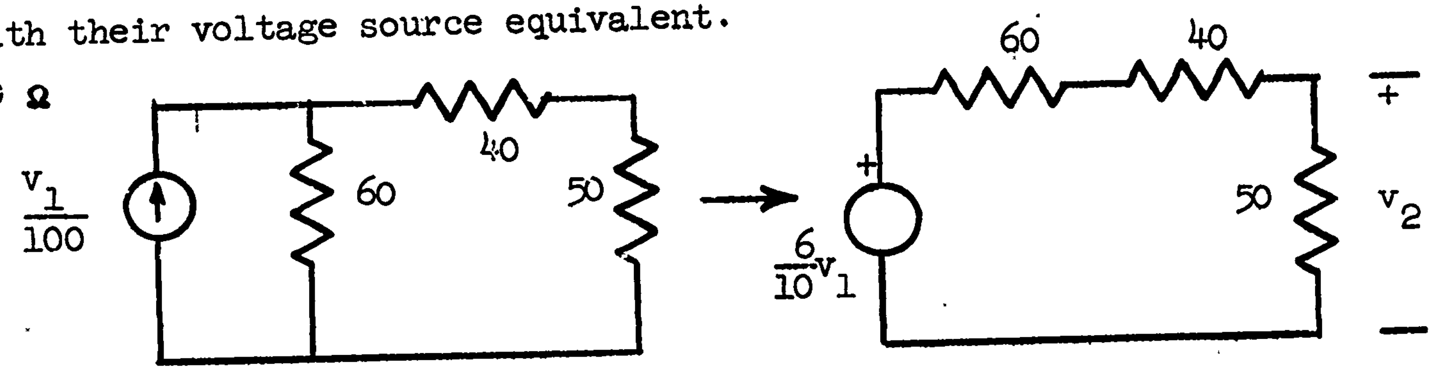
Suppose that the current source equivalent is used to replace v_1 and R_1. The result is shown in Fig. 12(b); it is neither a series nor a parallel circuit. Suggest a sequence of steps which will convert this network into a series network. Carry out these steps and sketch the resulting equivalent circuit, labeling all components.

52

Answer:

Combine R_1 in parallel with R_2 into an equivalent resistance; then replace this resistance and the $v_1/100$ current source with their voltage source equivalent.

$$R = \frac{100(150)}{250} = 60\ \Omega$$



We now have a simple series circuit from which the voltage v_2 can be easily calculated. State the value of v_2 in terms of v_1.

Answer:

$v_2 = .2v_1$

Did you recognize this structure as a voltage divider and use the voltage divider formula? If not, look back at the diagram and do it.

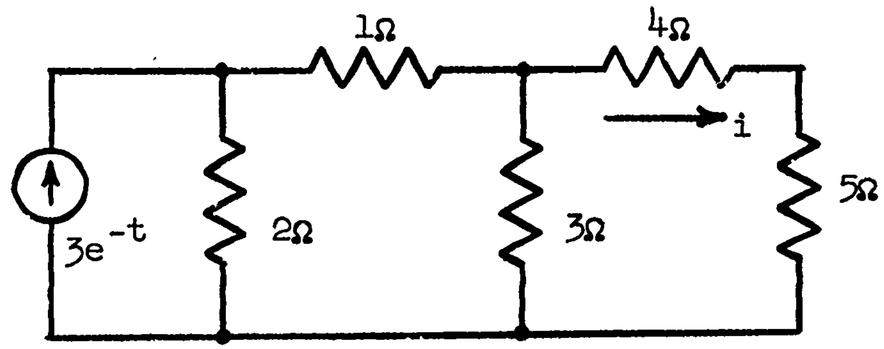


Figure 13

Before we attempt to generalize this result, let's take another example. It is desired to find the current i in the 4-ohm resistor in Fig. 13. Suggest a sequence of steps that converts this network to a series circuit containing the 4-ohm resistor. Carry out these steps and calculate i.

Answer:

First replace the current source and 2-ohm resistor with their voltage source equivalent. This leads to Fig. 13(a). Next combine the 2- and 1-ohm resistors; then replace the resulting 3-ohm resistor and the voltage source by their current source equivalent. This leads to Fig. 13(b). Next combine the two 3-ohm resistors in parallel; then replace the resulting 3/2 ohm resistor and the current source by their voltage source equivalent. This leads to Fig. 13(c). Finally, compute i:

$$i = \frac{3\,e^{-t}}{\frac{3}{2} + 4 + 5} = \frac{2}{7}\,e^{-t} \text{ amps.}$$

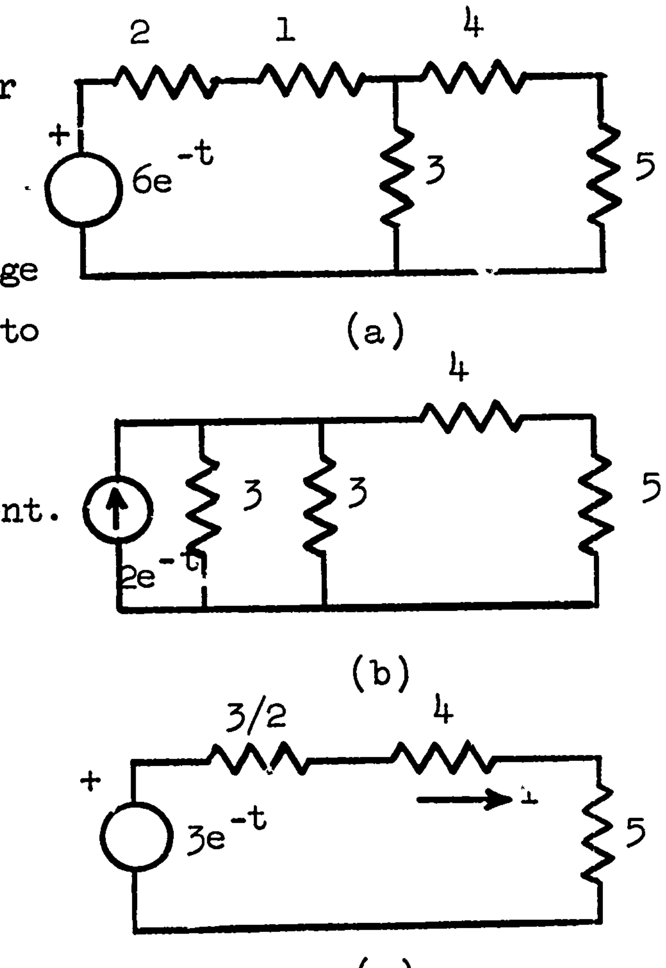
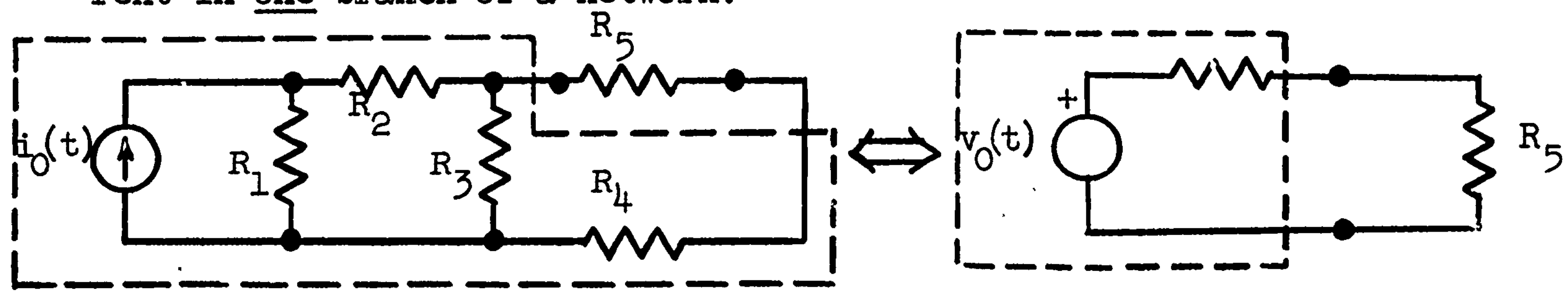


Figure 13

The preceding examples illustrate an important and rather general procedure. This procedure is most useful when it is desired to determine the voltage or current in <u>one</u> branch of a network:



The structure of the <u>rest</u> of the network is replaced by a <u>single equivalent</u>, consisting of a voltage source and a series resistor, which forms a single loop with the desired branch.

The equivalent is found by successive (one after the other) application of some simple steps. There are four basic steps that you have used to find the equivalent. Write them:

1. ____________________,
2. ____________________,
3. ____________________, and
4. ____________________.

Answer:

Rather than list these for you, it seems appropriate to assume that you wrote them correctly. Review the last few frames if you want to check yourself.

We will be discussing and using this procedure on the subsequent pages and at other times during this course.

This equivalent is so important and is used so often that it is given a name: it is called the <u>Thévenin equivalent</u>. That is, given a network containing resistors and voltage and current sources, with two terminals exposed as shown in Fig. 14, the <u>Thévenin equivalent</u> consists of ______________________________

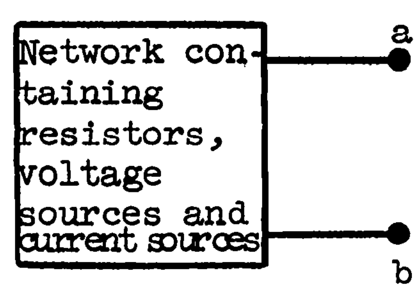
____

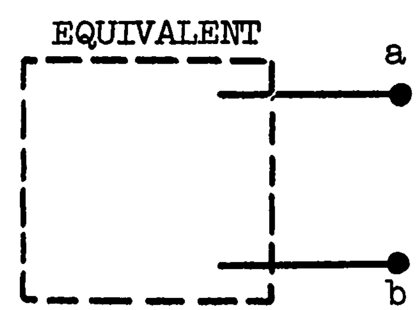
____

____

____.

(Now, draw the equivalent.)





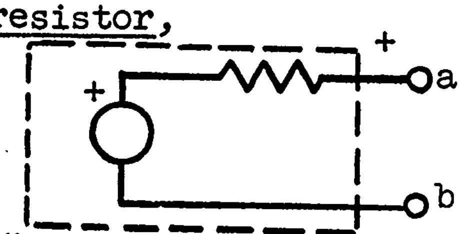
60

Answer:

the Thevenin equivalent consists of

(1)a voltage source in series with(2)a resistor,

(3)connected across those terminals.



(Your wording may be different but, again, your response must contain all three of the numbered items. If it does not, return and correct it.)

The equivalent, at a pair of terminals, of a network containing resistors, v- sources and i-sources which consists of a v-source in series with a resistor is called the ________________ equivalent. (Write it. -- Say it out loud. You will be using it many times.)

62

Answer:

<u>Thévenin</u>.

(Remark: This was named after a Frenchman, not a Russian, as one might suspect from the name.)

As you might suspect, the equivalent voltage and resistance in the <u>Thévenin</u> equivalent of a two-terminal network are called the ________________ ________________ voltage and the ________________ ________________ resistance, respectively.

Answer:

<u>Thévenin equivalent</u>

<u>Thévenin equivalent</u>

<u>Remark</u>:

Up to this point we have discussed one method of determining the Thévenin equivalent of a given network. This method consists of the successive use of a v-source equivalent, an i-source equivalent, series combinations of resistors and parallel combination of resistors. For our purposes, the present method is the only one you will need. In a later unit, an alternative method will be described.

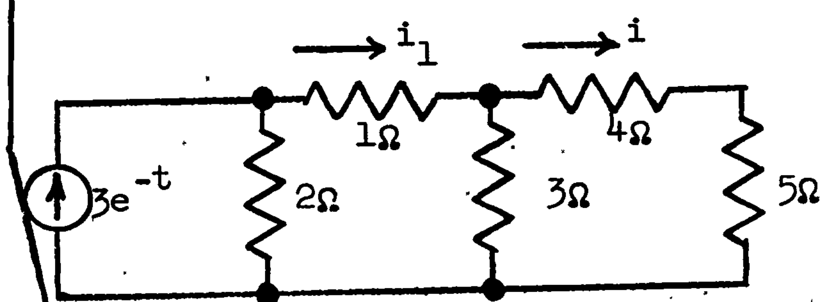


Figure 13

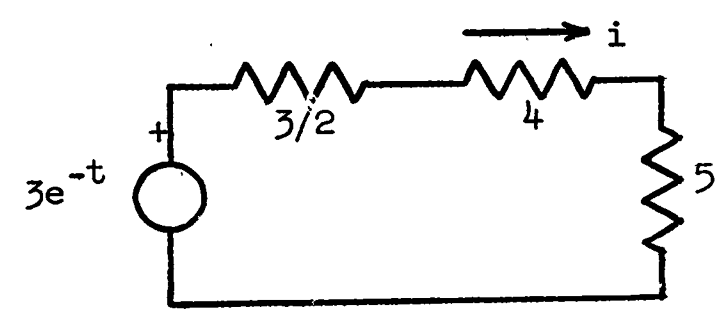


Figure 14

Now suppose that it is desired to determine the voltage or current or power dissipated in <u>more than one</u> branch of a network. For example, in Fig. 13 (repeated) suppose that in addition to $\underline{i}$ it is desired to find the power dissipated in the 1-ohm resistor. (It is sufficient to find the current $\underline{i}$, since once it is found, the desired power will follow.)

In carrying out the suggested procedure for the determination of $\underline{i}$, the structure of the network is destroyed, so that in the resulting equivalent network (repeated as Fig. 14) the 1-ohm resistor does not even show up. It appears that we are stymied. However, once $\underline{i}$ has been determined from Fig. 14, we can return to the original network in Fig. 13 in which $\underline{i}$ is now known. By applying Kcl at the junction of the 1-, 3- and 4-ohm resistors, $\underline{i}_1$ is determined in terms of $\underline{i}$ and the current in the 3-ohm resistor which, by Ohm's law is 1/3 the voltage across it. It remains to determine this voltage. By applying Kvl, express the voltage across the 3-ohm resistor in terms of known quantities. (Remember, $\underline{i}$ was found to be $\frac{2}{7}e^{-t}$.)

Answer:

By Kvl the voltage across the 3-ohm resistor is equal to the sum of the voltages across the 4- and 5-ohm resistors. Since the current in these resistors is known, this voltage can be found. Thus,

$$v \text{ (across 3-ohm resistor)} = 4i + 5i = \frac{18}{7} e^{-t}$$

Finally, the current i_1 and the power dissipated in the 1-ohm resistor are:

$i_1 =$

$p =$

68

Answer:

$$i_1 = i + \frac{1}{3} \text{ voltage across 3-ohm resistor} = \frac{8}{7} e^{-t}$$

$$p = i_1^2 = \frac{64}{49} e^{-2t}$$

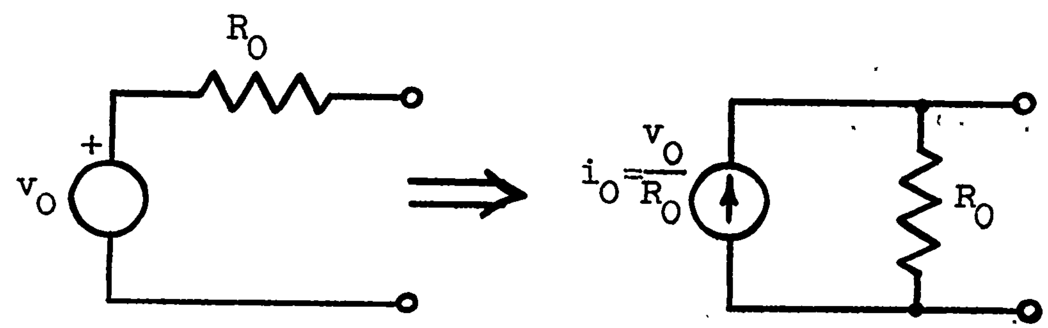
A rather general method of solution, then, involves replacing a <u>sub-network</u> by its Thévenin equivalent, solving for a voltage or current in a given branch, then using this solution, together with known sources, in the <u>original</u> network to find other branch voltages or currents.

The Thévenin equivalent is simply a voltage source in series with a resistor. But we have previously found that equivalent to a v-source of voltage v_0 in series with a resistor R_0 is another network consisting of ____. (Also draw it.)

70

Answer:

a current source $i_O = v_O/R_O$ in parallel with the resistor R_O.



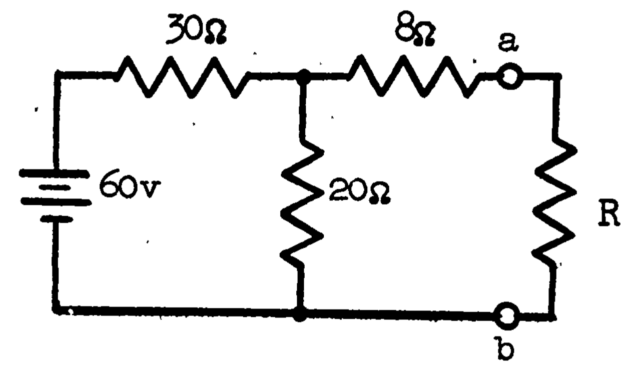


Figure 15

Since the v-source and series resistor combination is equivalent to the original subnetwork, the i-source and parallel resistor combination is also equivalent to the subnetwork. This combination is also given a name; it is called a <u>Norton equivalent</u>.

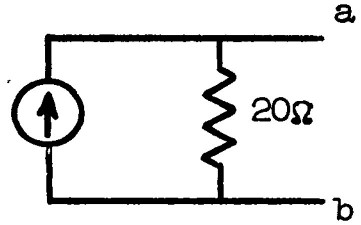
In the solution procedure we have been discussing, it may sometimes be more convenient to arrive at a final network which is a <u>parallel</u> combination of branches containing the branch in question, rather than a <u>series</u> circuit. This was, in fact, done on page 37. (Read that page and the answer again before going on.) Both the Thevenin equivalent and the ____________ equivalent are tools to be used where each is most appropriate.

Draw the Norton equivalent of the subnetwork to the left of terminals a-b in Fig. 15 and show the correct numerical values.

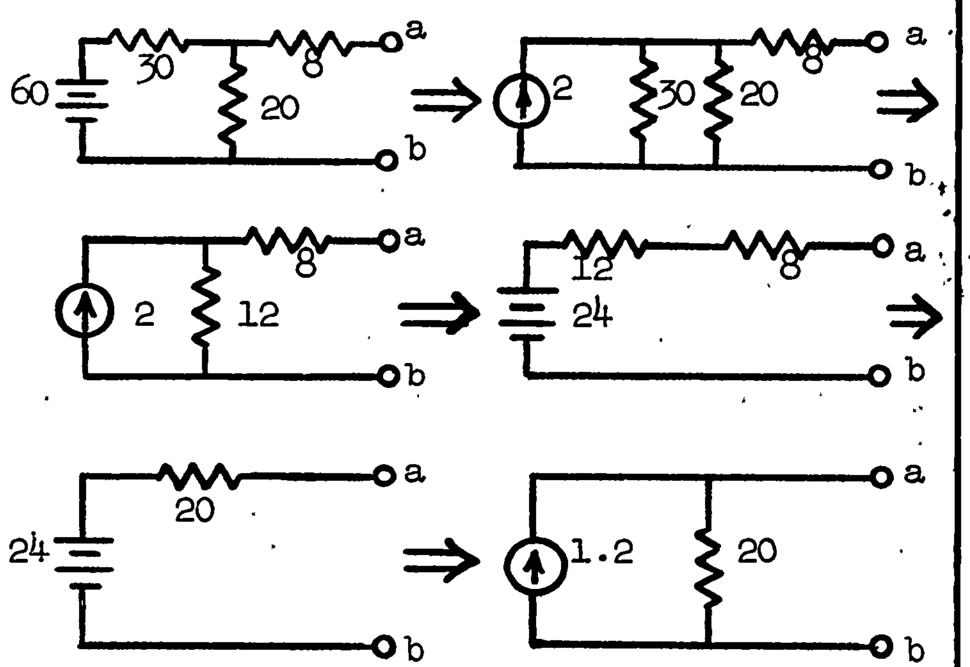
72

Answer:

1.2amp.



Obtained by:



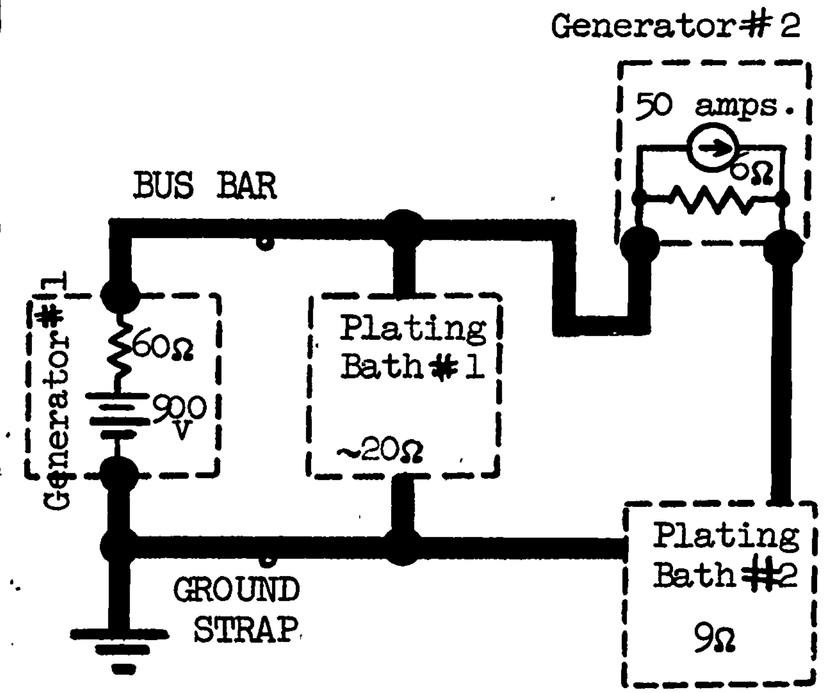
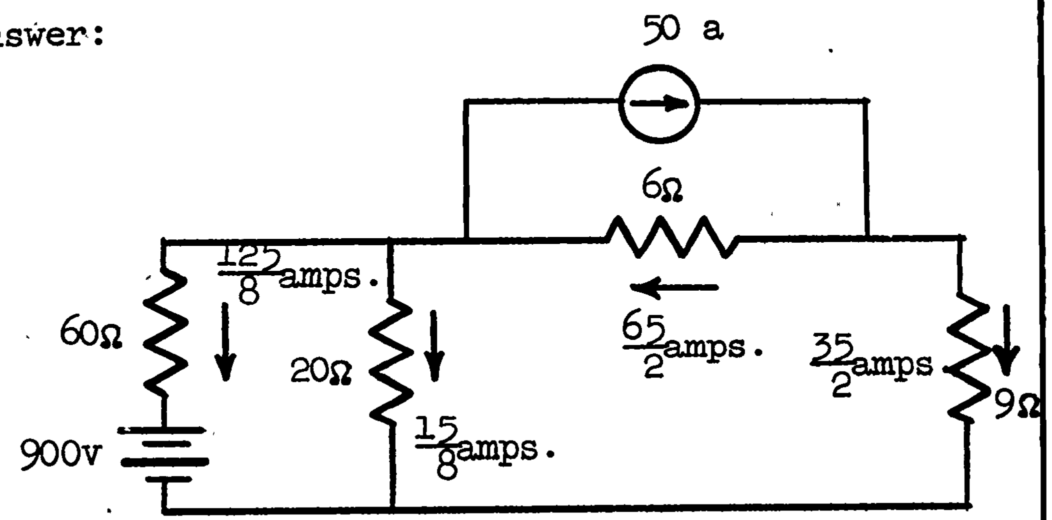


Figure 16

Knowing that you are studying engineering, the foreman of a small electroplating shop has asked you for help. He is blowing fuses and cannot figure out what his peak currents should be.

After some measurements, you were able to make a reasonable guess at his loads and sources and arrived at Fig. 16. What currents should he expect in the branches of his system? Assume that each plating bath can be considered equivalent to an ideal resistor. Determine the currents in more than one way to check yourself. Focus initially on one branch, replacing the remaining network by an appropriate equivalent, then returning to the original network. Then choose another branch on which to focus.

Answer:



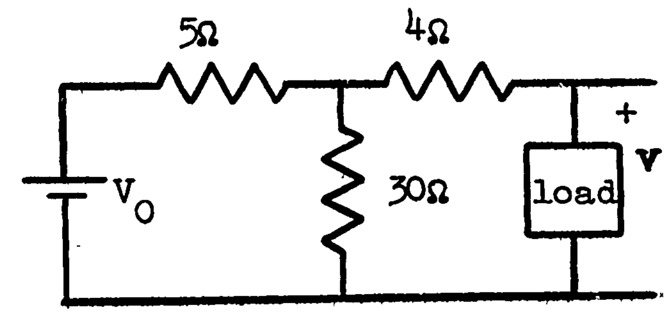


Figure 17

Let's now consider another class of problems, one in which the desired current or voltage or power in one branch of a network is specified and it is desired to determine the required source voltage. Figure 17 shows such a case, in which it is desired to supply 500 watts of power to a resistive load across which the voltage is to be 100 volts. It is required to find the source voltage V_0.

This problem has features similar to the type of problem we have already solved. Instead of first having solved for a branch voltage or current, by using a Thevenin equivalent, and then returning to the original network, a branch voltage or current is already given. The remaining procedure, then, should be the same; namely, the alternate use of Kvl, Kcl and Ohm's law.

Use this procedure to find the required value of V_0.

76

Answer:

$V_0 = 165$ volts

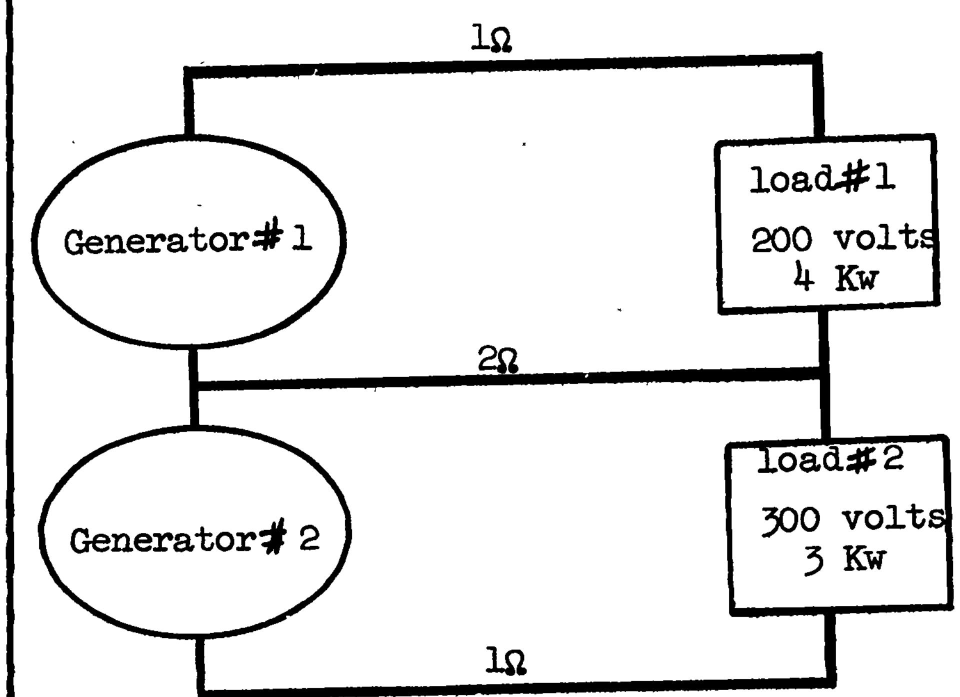


Figure 18

Figure 18 illustrates a 3-wire power distribution system. Dc power is to be supplied to two loads as indicated. The transmission system has losses which can be represented by the resistances shown. Two dc generators are available whose voltages can be set over the following ranges:

Generator 1: from 200 to 250 volts

Generator 2: from 275 to 300 volts

Will these two generators be adequate to supply the required loads? If so, at what values should their voltages be set? Compute also the power that each generator will supply. You can detect possible numerical errors by verifying that the power supplied by the generators equals the power lost plus the power delivered to the loads.

78

Answer:

The generator will be adequate.

Set the voltages at:

Generator 1: 240 volts; power supplied = 4.8 Kw

Generator 2: 290 volts; power supplied = 2.9 Kw

<u>Summary</u>:

1. A network containing resistors, v-sources and i-sources can be replaced at a pair of terminals by a <u>Thévenin</u> equivalent consisting of ________________________________.
2. Another structure equivalent to this is called a ______________ ______________ and it consists of ________________________________.
3. The Thévenin or Norton equivalent of a given network can be determined by successively converting from a v-source equivalent to ________________ while combining resistors in __________ and ______________.
4. Once a branch current or voltage is determined, or when a required value of branch current or voltage is specified, it is possible to work through the rest of the network to find other voltages and currents by alternate applications of ______, __________ and ______________.

Answer:

1. Thévenin's equivalent consists of a voltage source in series with a resistance.
2. Norton equivalent consists of a current source in parallel with a resistance.
3. from a v-source equivalent to an i-source equivalent while combining resistors in series and in parallel.
4. by alternate applications of Kvl, Kcl and Ohm's law.

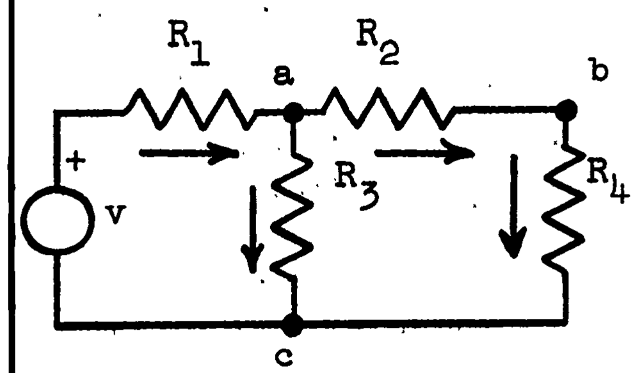


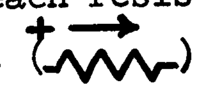
Figure 19

Section 3

The method of solution of network problems discussed in the last section proceeds by replacing, in simple stages, the structure of a subnetwork by a simple equivalent, so that a single equation can be written for the only remaining variable, and solved. The validity of the procedure is established by applying the basic relationships -- Ohm's law and Kirchhoff's two laws.

An alternative procedure is to apply these basic laws to a network without replacing any part of it by an equivalent and thereby changing its structure. The detailed development of such a procedure is the subject of this section.

The case of a series circuit (or a parallel combination) is simple enough that no further consideration will be given to them. The introductory discussion will be carried on in terms of the network shown in Fig. 19. Although

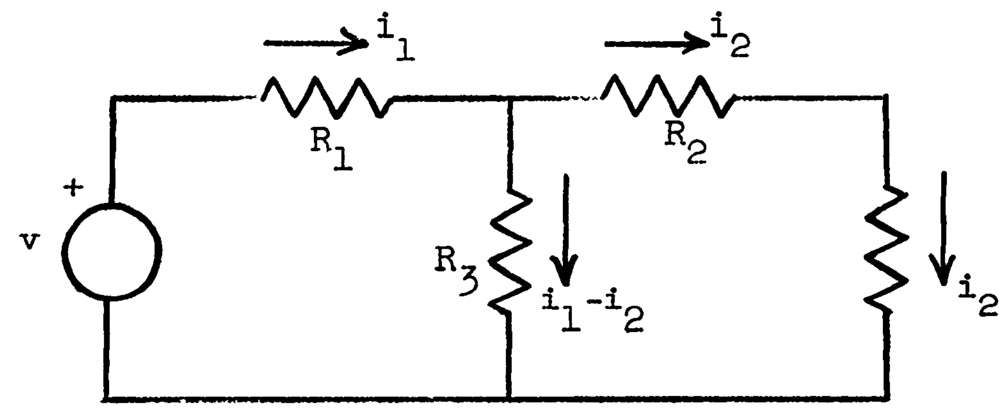
branch voltage and current references are arbitrary, let us choose the references for each resistor such that the voltage plus is at the tail of the current arrow. () Thus, Ohm's law will always be $v = + Ri$. Then only one of the references (plus sign or arrow) is sufficient to designate both references. In Fig. 19 an arrow is used.

In the three-step procedure to be described, now, we will apply first Kcl, then Kvl, and finally Ohm's law.

The first step is to apply Kcl. Draw your own diagrams of the same circuit and let i_1 and i_2 be the currents in R_1 and R_2. Applying Kcl at the nodes, write the other two branch currents in terms of i_1 and i_2 alongside the reference arrows in your diagram.

84

Answer:



The next step is to apply Kvl.

Write Kvl equations around the two inner "meshes" of the network in terms of voltages (that is, without applying Ohm's law).

86

Answer:

$$- v + v_1 + v_3 = 0$$

$$- v_3 + v_2 + 4_3 = 0$$

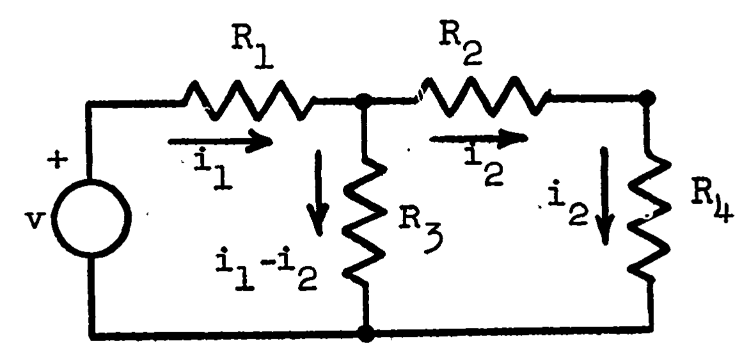


Figure 19 (repeated)

There is one additional closed path but the equation obtained by writing Kvl around this path is the sum of the other two equations, and hence, not independent. (Verify this.)

We have used Kcl and Kvl. All that is left of the basic equations is Ohm's law. Using Ohm's law, replace all the voltages in the Kvl equations with appropriate expressions containing the previously labeled branch currents. Finally, collect terms and transpose appropriately.

Answer: $- v + R_1 i_1 + R_3(i_1 - i_2) = 0$

$- R_3(i_1 - i_2) + R_2 i_2 + R_4 i_2 = 0$

Finally,

$$(R_1 + R_3) i_1 - R_3 i_2 = v$$

$$- R_3 i_1 + (R_2 + R_3 + R_4) i_2 = 0$$

This is a set of two simultaneous linear equations in two unknowns and can be solved by the methods of algebra. They are called the <u>loop equations</u> from the fact that their basic form is that of Kirchhoff's voltage law written around closed paths, or loops.

For convenience, let the following numerical values be given:

$R_1 = 1$, $R_2 = 2$, $R_3 = 3$ and $R_4 = 4$, all in ohms; and $v = 10 \sin 3t$. With these values the equations become

$$4i_1 - 3i_2 = 10 \sin 3t$$

$$-3i_1 + 9i_2 = 0$$

Solve for i_1 and i_2 in any way that you can.

90

Answer:

$$i_1 = \frac{90}{19} \sin 3t$$

$$i_2 = \frac{30}{19} \sin 3t$$

(If you are not able to solve linear simultaneous equations, do not go on until you have gotten help.)

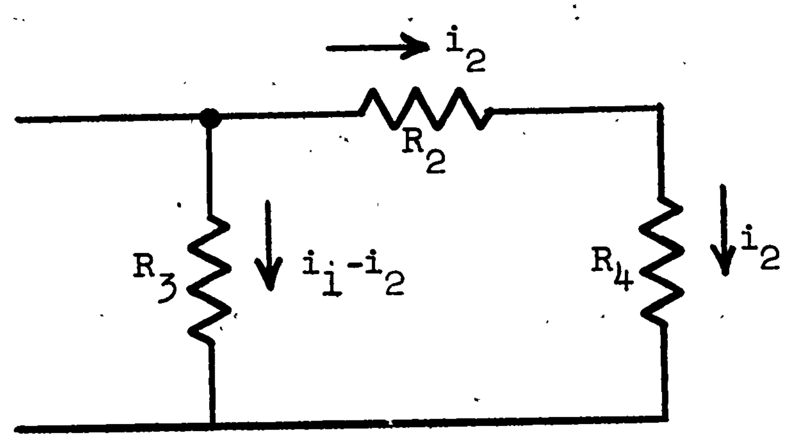


Figure 20

In the example we just completed, after writing Kvl in terms of voltages, we then applied Ohm's law. This last is such a simple step that it can be combined with the previous one. Thus, when writing Kvl, instead of <u>writing</u> a voltage, we <u>think</u> a voltage but write it Ri with the appropriate R and i. Thus, in Fig. 20, go clockwise around the loop starting with R_2 and write Kvl immediately in terms of the currents.

92

Answer:

$$R_2 i_2 + R_4 i_2 - R_3(i_1 - i_2) = 0$$

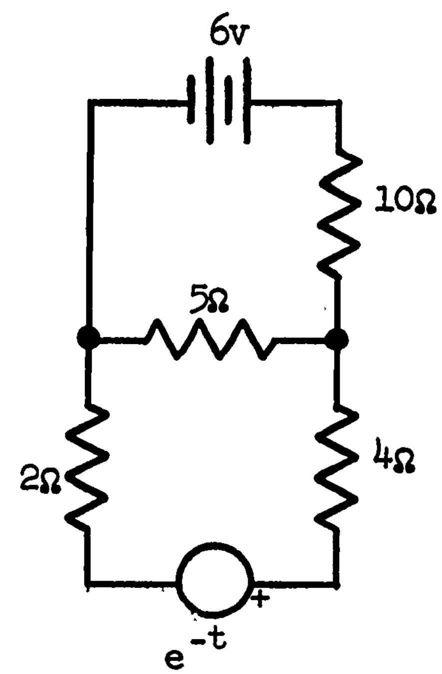
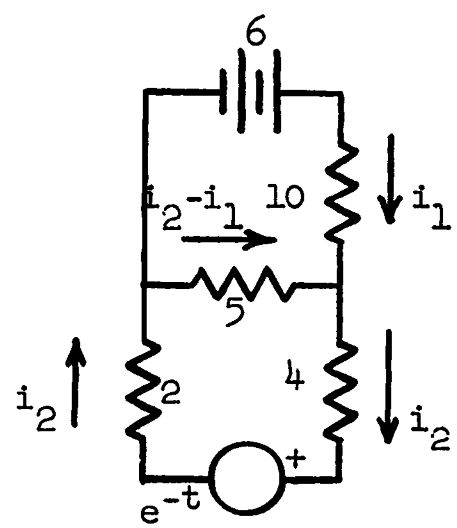


Figure 21

The process of arriving at the loop equations is straightforward. Given a network, we first draw references for all the branch currents and express the branch currents in terms of some of them by applying Kcl. Then we apply Kvl to get a set of independent equations, while simultaneously using Ohm's law, so that the variables in these equations are currents -- those branch currents in terms of which all branch currents are expressed.

Test out this approach on Fig. 21 and write a set of loop equations for the network shown.

Answer:



$$-6 + 10i_1 - 5(i_2-i_1) = 0$$

$$5(i_2-i_1) + 4i_2 + e^{-t} + 2i_2 = 0$$

This is just one possibility. Your answer can differ from this in two possible ways: (1) It is possible to choose another set of currents rather than the ones chosen here. Thus, the current in the 5-ohm resistor could be called i_3 and the ones labeled i_2 could be expressed as $i_1 + i_3$. (2) It is possible to write Kvl around one of the "meshes" and the outside loop, rather than the two meshes as done here.

This is a set of two equations in two unknowns which can be solved by the rules of algebra. (We won't carry the solution through here.)

The currents in terms of which all the other currents are expressed are called the loop currents.

In Fig. 22, express the remaining branch currents in terms of the ones labeled i_1, i_2 and i_3, which are ______ ________.

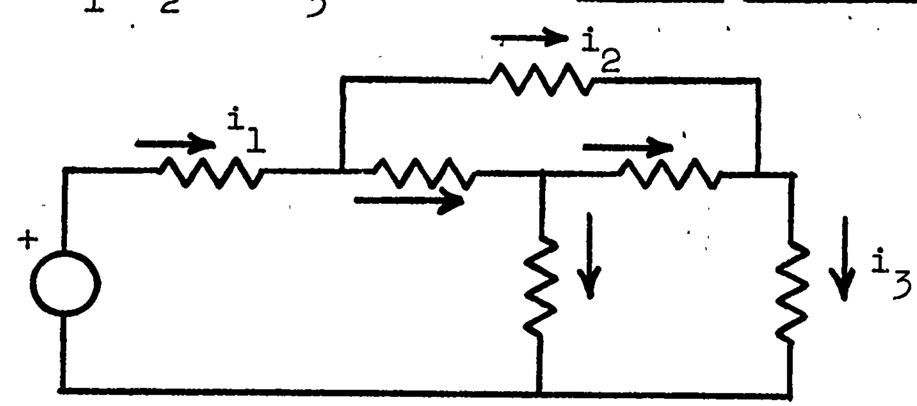
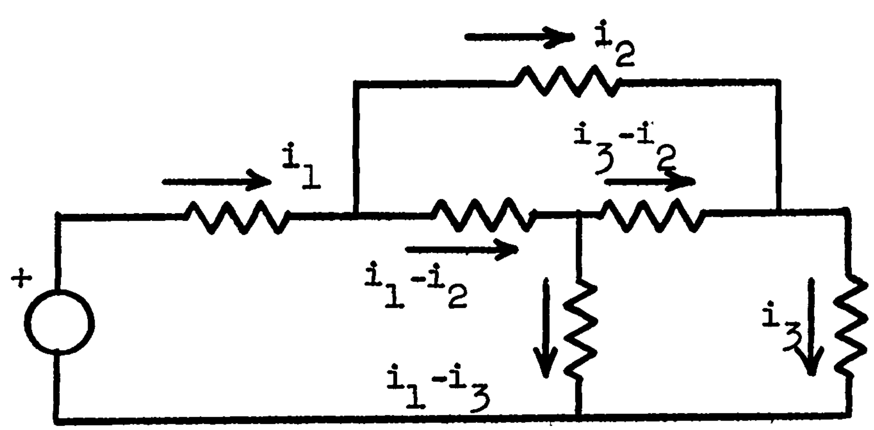


Figure 22

Answer:

i_1, i_2 and i_3 are <u>loop currents</u>.



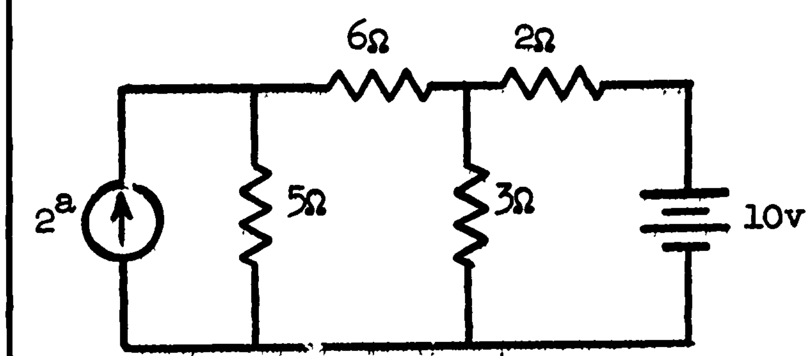
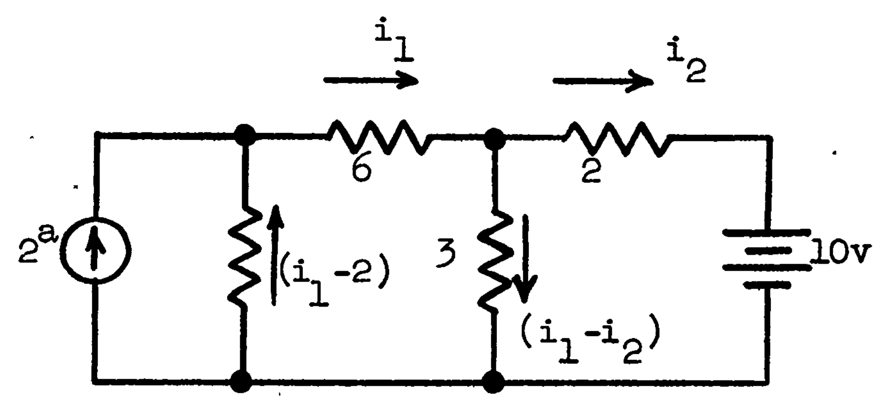


Figure 23

Let us consider just one more point. In this discussion of loop equations, there has been no current source. Figure 23 shows an example of a network containing a current source. By following the process under discussion, write a set of loop equations. Note whether there are any special difficulties with the current sources. (If you wish to compare your answer with the one to be given, choose as loop currents those in the 6- and 2-ohm resistors.)

Answer:



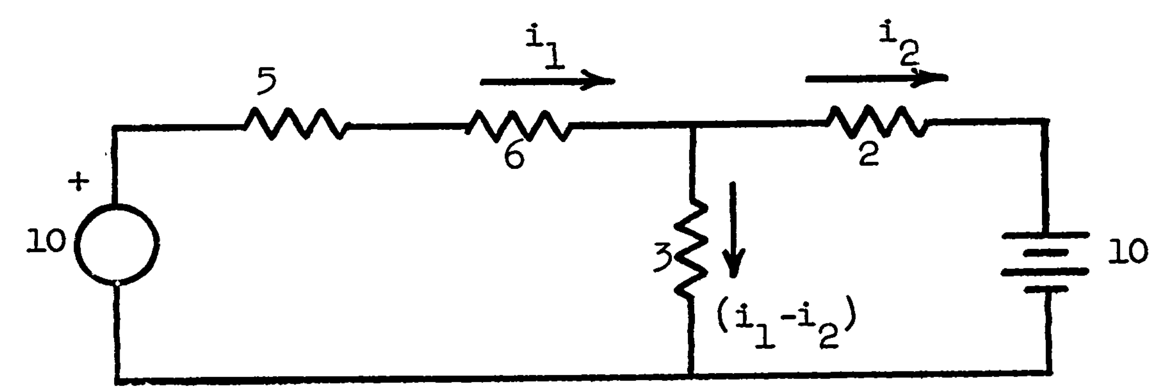
$5(i_1-2) + 6i_1 + 3(i_1-i_2) = 0$ or $14i_1 - 3i_2 = 10$

$-3(i_1-i_2) + 2i_2 + 10 = 0$ $\quad -3i_1 + 5i_2 = -10$

No difficulty is introduced by the presence of the current source.

An alternative procedure for handling current sources when carrying on a loop analysis is to convert to a v-source equivalent. Thus, in the example under consideration, suppose the current source and parallel 5-ohm resistor are converted to a v-source equivalent. Draw the resulting network and write the loop equations choosing the same loop currents as before.

Answer:



$$-10 + 5i_1 + 6i_1 + 3(i_1 - i_2) = 0 \qquad \text{or} \qquad 14i_1 - 3i_2 = 10$$

$$-3(i_1 - i_2) + 2i_2 + 10 = 0 \qquad\qquad -3i_1 + 5i_2 = -10$$

These are the same equations as before. The solution is $i_1 = 20/61$ and $i_2 = -110/61$ amps. (Check your own solution against this.)

Once the loop currents are determined, all other branch currents will become known since they are expressed in terms of the loop currents. By Ohm's law, all the branch voltages can also be found. Finally, the power dissipation in any branch or delivered by any source can be calculated.

In the example just completed, determine the power <u>supplied</u> by each of the two sources in the original network of Fig. 23.

Answer:

Power supplied by battery $= -10i_2 = \frac{1100}{61}$ watts.

Power supplied by current source $= \frac{1020}{61}$ watts.

(The voltage across the current source is $5(i_1 - 2) = -\frac{510}{61}$ volts with the reference at the tail of the current arrow. Hence, the power <u>supplied</u> is $-(2)(-\frac{510}{61}) = \frac{1020}{61}$ watts.)

This completes the discussion of <u>loop equations</u>. But there is still <u>another</u> way we can approach the analysis of problems like this.

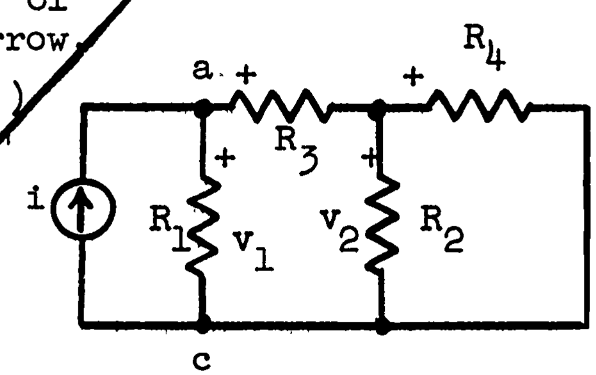


Figure 24

In considering another way of combining the three basic laws (Kcl, Kvl and Ohm's law) let us use, for an initial example, the network of the last example, but without the battery and without numerical values, as shown in Fig. 24. (The same convention is used about current and voltage references, but this time a plus sign is shown.)

We started the discussion of loop equations by applying Kcl and thereby expressing all the branch currents in terms of just some of them. Why not do it the other way and apply Kvl first? Then express all the branch voltages in terms of just some of them.

Let v_1 and v_2 be the voltages across R_1 and R_2. Apply Kvl as needed and write the other branch voltages in terms of v_1 and v_2.

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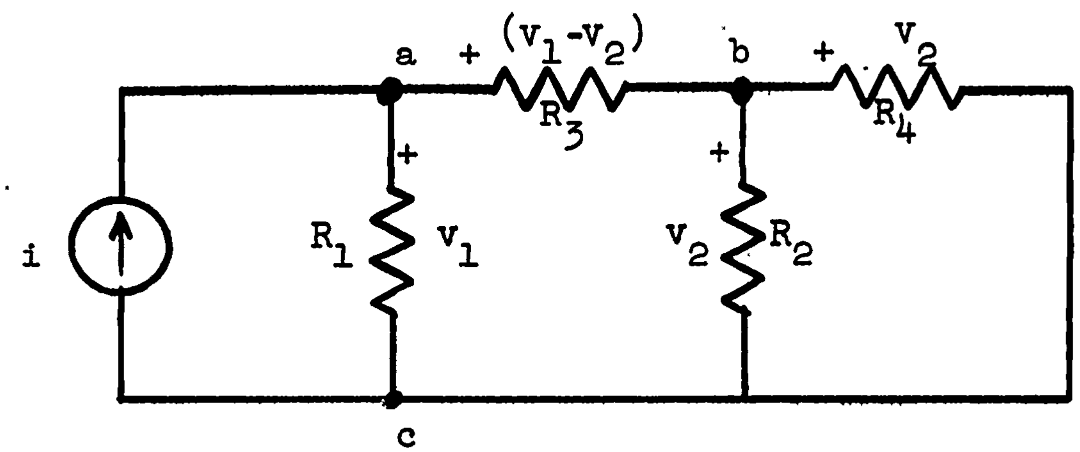
Answer:



Next, apply Kcl at the nodes. But now that you have some practice, you can simultaneously insert Ohm's law so that the resulting equations are in terms of voltages.

Write the equations at nodes a and b. (If you want to check with the answers to be given, use leaving as the junction reference.)

Answer:

$$- i + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_3} = 0$$

$$- \frac{(v_1 - v_2)}{R_3} + \frac{v_2}{R_2} + \frac{v_2}{R_4} = 0$$

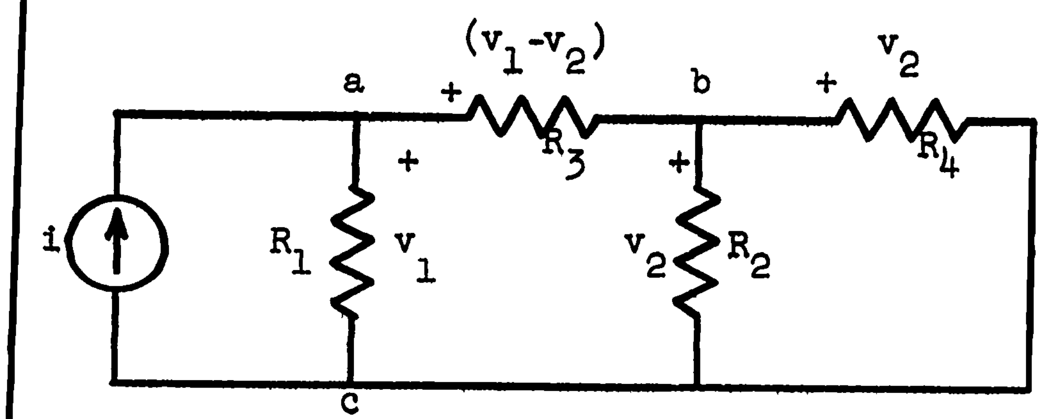


Figure 24 (repeated)

This is a set of two independent equations in two voltage variables. (The source current i is presumed to be known.) From this point on, the solution is a matter of algebra.

Note from Fig. 24 that the branch voltages v_1 and v_2 are also the voltages of nodes a and b relative to that of node c. We can look upon c as a datum node to which the voltages of all other nodes are referred. These voltages are then called the node voltages. The equations that result when Kcl is applied at all the nodes other than the datum node are called the node equations.

Let us now restate the steps in the procedure for arriving at the node equations. First, choose any one node as a ____________ node. Then, label all other node voltages (v_1, v_2 or v_a, v_b, etc.). These are the voltages of the nodes relative to that of the datum node. Since each branch lies between two nodes, each branch voltage will equal the difference of two ________ ________. (The branch voltage of a branch connected to the datum node is just one node voltage, since the voltage of the datum node is zero.) Expressing the branch voltages in this form amounts to an application of Kvl. The next step is to

apply ________________ at each node but the datum node. Simultaneously, Ohm's law is applied, so that the terms as written are in terms of voltages. The resulting equations are the ______________ ________________.

Answer:

Most probably you do not need to be reassured of the correctness of your responses in filling in the blanks.

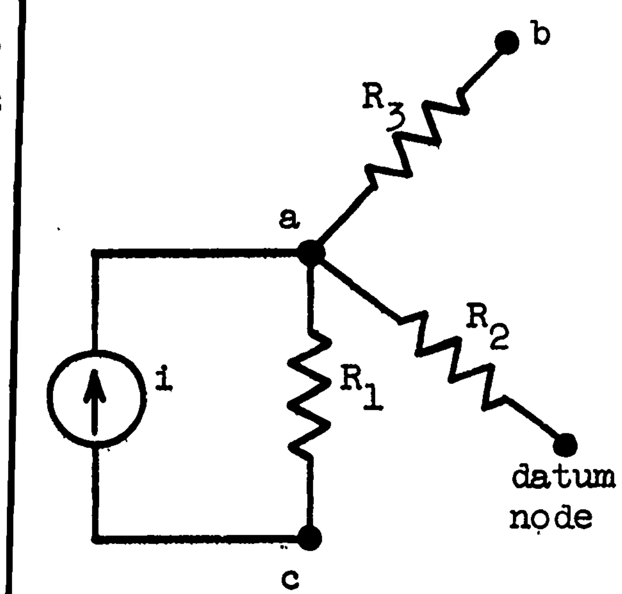


Figure 25

Refer to Fig. 25 which shows a partial network.

When applying Kcl to arrive at the node equation for node a, we say to ourselves something like this: "The current leaving node a through the current sources is $-i$; the current leaving node a through R_1 equals the branch voltage divided by R_1, which is $(v_a - v_c)/R_1$; the current leaving node a through R_2 equals the branch voltage divided by R_2, which is ________; the current leaving node a through R_3 equals ______________________.
The sum of all these is zero." Thus, for node a the node equation is ______________________.

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Answer:

the node equation is

$$-i+\frac{v_a - v_c}{R_1}+\frac{v_a}{R_2}+\frac{v_a - v_b}{R_3}=0$$

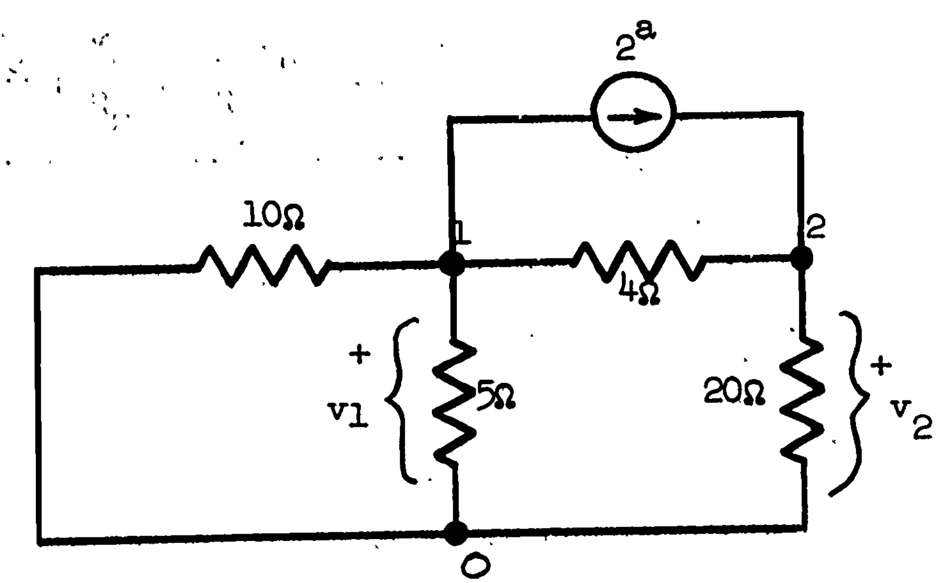

Figure 26

Let's now practice writing node equations on the network in Fig. 26. Choose node 0 as a datum node and let v_1 and v_2 be the <u>node voltages</u> of the other two nodes. The references of these voltages are implicit; that is, the plus signs should be at the <u>non</u>-datum nodes. For clarity, they have been shown explicitly in Fig. 26. Write the node equations; then solve for the node voltages.

Answer: These are the node equations:

node a: $$\frac{v_1}{10} + \frac{v_1}{5} + \frac{v_1 - v_2}{4} + 2 = 0$$

node b: $$\frac{v_2 - v_1}{4} + \frac{v_2}{20} - 2 = 0$$

The solution is: $$v_1 = -\frac{40}{41} \text{ volts}$$

$$v_2 = \frac{240}{41} \text{ volts}$$

Once the node voltages have been found, all the branch voltages become known, since they are simply differences of node voltages. From Ohm's law, then, all the branch currents can be found. With currents and voltages known, any desired power can be calculated.

Find the power p supplied by the current source in Fig. 26.

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Answer: $p = \frac{560}{41}$ watts.

The voltage across the current source, with the reference such that the power supplied is positive is $v_2 - v_1 = 280/41$, giving the stated power.

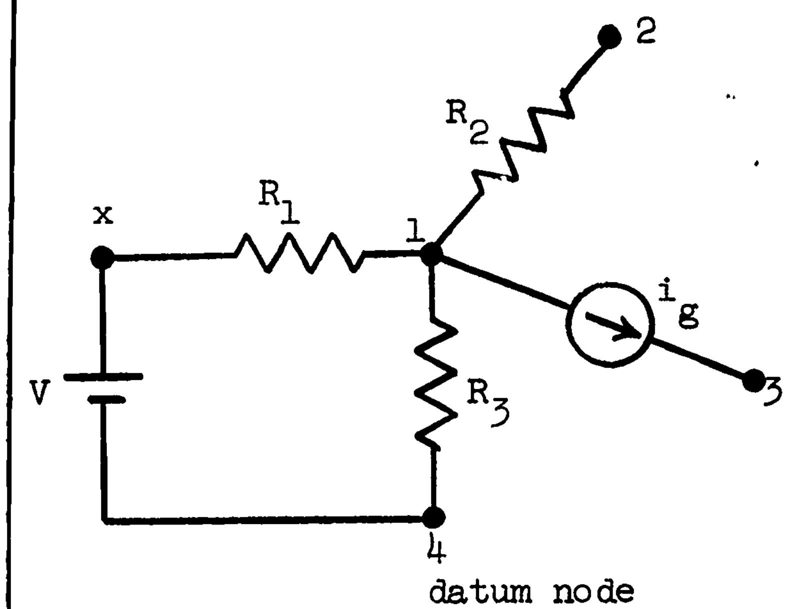


Figure 27

When there is a voltage source in a network, no insurmountable problem is encountered in writing node equations. Consider the partial network shown in Fig. 27. Let one terminal of the voltage source (battery) be chosen as the datum node. The voltage of the point marked "x" relative to the datum node is known; it is the battery voltage V.

To write the node equation at node 1, we apply Kcl and set the sum of all currents leaving node 1 equal to zero, while using Ohm's law to write each current in terms of voltages. Thus,

current leaving node 1 through R_2 = ____________

current leaving node 1 through current source = ____________

current leaving node 1 through R_3 = ____________

current leaving node 1 through R_1 = ____________

Answer:

current leaving node 1 through R_2 $= \frac{v_1 - v_2}{R_2}$ amps.

current leaving node 1 through current source $= i_g$ amps.

current leaving node 1 through R_3 $= \frac{v_1}{R_3}$ amps.

current leaving node 1 through R_1 $= \frac{v_1 - 6}{R_1}$ amps.

The branch voltage across R_1 equals v_1 minus the voltage of point "x", which is 6 volts.

Even if one of the battery terminals is not chosen as a datum, node equations are easily written. Instead of node 4, let node 3 be chosen as a datum. The voltage of point "x" relative to the datum node equals its voltage relative to node 4 plus the voltage of node 4 relative to that of the datum. (If this is not readily apparent to you, draw a separate diagram, placing the points "x", 4 and 3 on it. Then write Kvl around an imaginary closed path through these points.)

Write the node equation at node 1 using node 3 as datum.

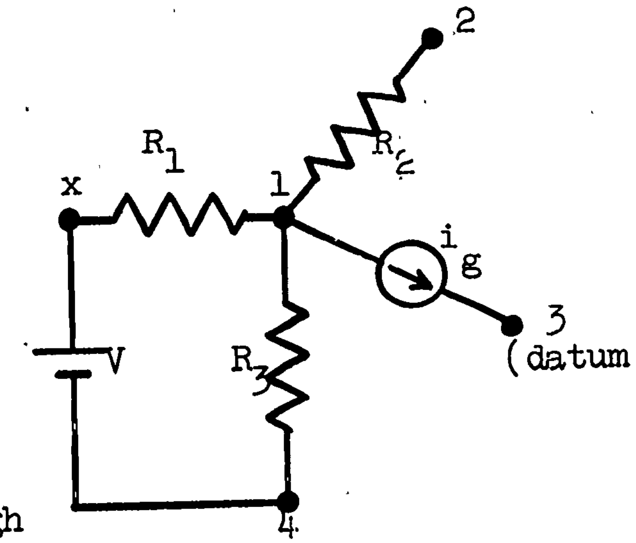
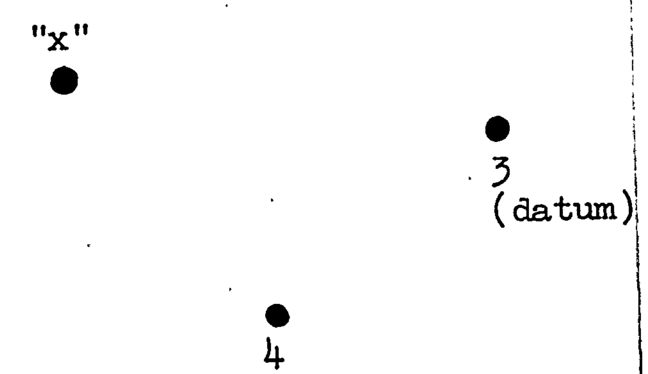


Figure 28



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Answer:

$$\frac{v_1 - v_2}{R_2} + i_g + \frac{v_1 - v_4}{R_4} + \frac{v_1 - v_4 - V}{R_1} = 0$$

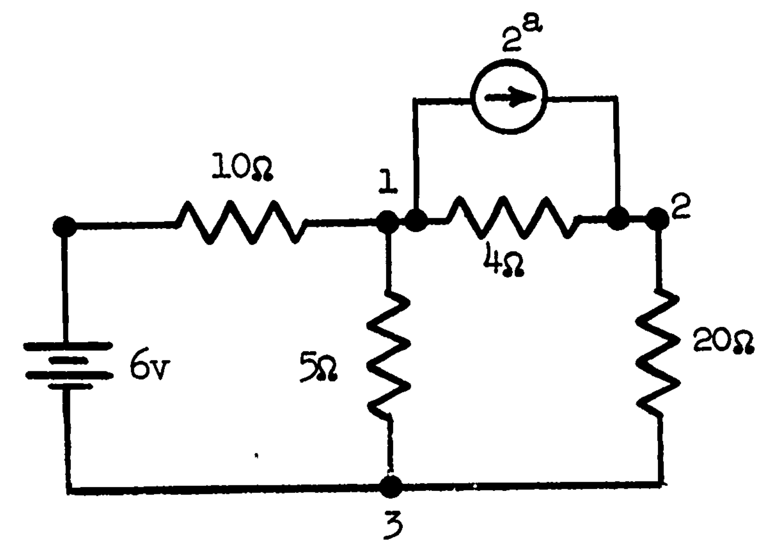

Figure 29

Another way to handle the presence of a voltage source when carrying on a node analysis is to convert it to an i-source equivalent. In Fig. 29, replace the battery and series 10-ohm resistor by an i-source equivalent and write the node equation at node 1, choosing node 3 as datum. Also, write the node equation at node 1 without this conversion and compare the two.

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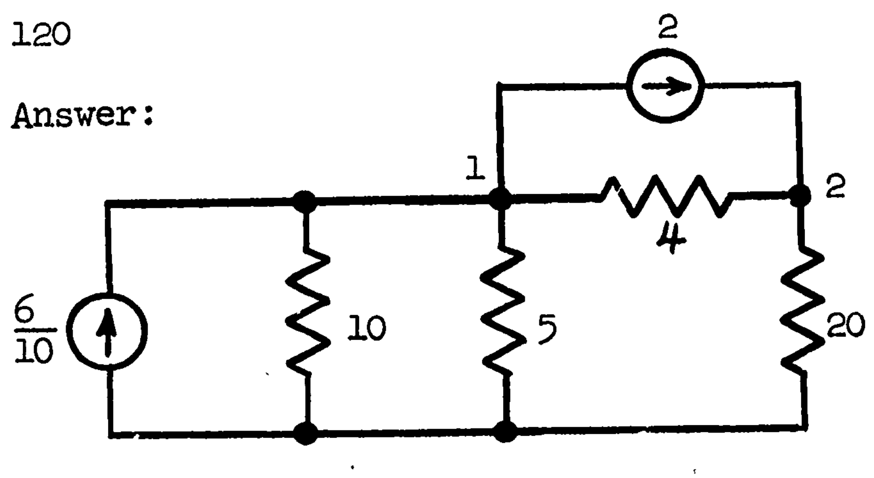
Answer:



$$-\frac{6}{10}+\frac{v_1}{10}+\frac{v_1}{5}+\frac{v_1-v_2}{4}+2=0$$

Same equation obtained without converting.

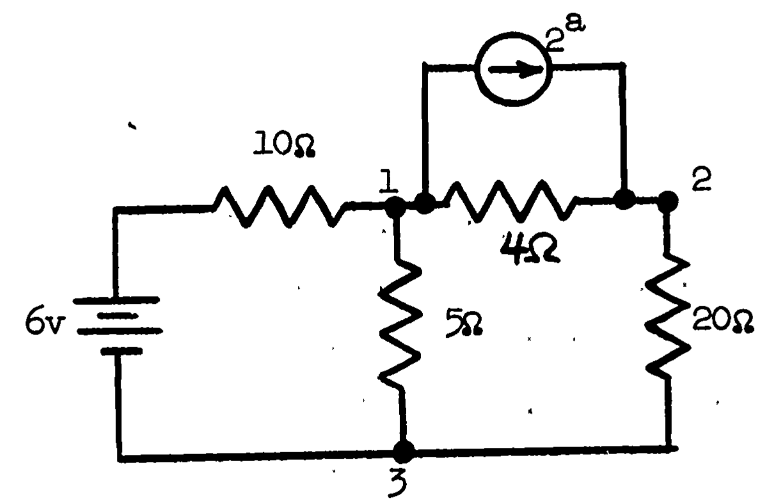


Figure 29 (repeated)

Complete writing the node equations in Fig. 29 and solve them. Finally, compute the power supplied by the battery in Fig. 29.

Answer:

$$-\frac{6}{10}+\frac{v_1}{10}+\frac{v_1}{5}+\frac{v_1-v_2}{4}+2=0$$

$$-2+\frac{(v_2-v_1)}{4}+\frac{v_2}{20}=0$$

or

$$11v_1 - 5v_2 = -28$$

$$-5v_1 + 6v_2 = 40$$

whose solution is $v_1 = \frac{32}{41}$ volts

$v_2 = \frac{300}{41}$ volts

The current in the 10-ohm resistor with reference toward node 1 is $\frac{6-v_1}{10} = \frac{107}{205}$ amp.

The power supplied by the battery is $6\left(\frac{107}{205}\right) = \frac{642}{205}$ watts.

This completes the discussion of node equations.

<u>Superposition</u>

We have been considering a number of procedures whereby electric networks consisting of resistors and sources can be analyzed. In all cases, the fundamental expressions relating the voltage and current variables are Kirchhoff's laws and Ohm's law. Because of the nature of the equations that result from these relationships, a very important observation follows.

In a network, it is of interest to determine whether the current or voltage in a branch due to all the sources in the network can be obtained by finding the current or voltage resulting from the presence of each source alone, then adding these. A similar question arises in many other areas of science and engineering; it is answered by the <u>principl of superposition</u> which can be stated in a very general way as follows:

Whenever an effect is linearly related to its cause, then the effect owing to a combination of causes is the same as the sum of the effects owing to each cause acting alone, all other causes being inoperative, or deactivated.

In the case of an electric network, the effects are currents and voltages in the branches of the network and the causes are the sources. Ohm's law is a linear relationship expressing the fact that the voltage and current of a branch are proportional: if the current is doubled, the voltage will double. Kirchhoff's laws simply add a number of terms together, with the result that the basic linear relationship is retained. It follows that the principle of superposition applies in the analysis of linear electric networks.

It remains to interpret what it means to say that a cause is inoperative or deactivated. In our case, the causes are voltage sources and current sources. Remembering the definition of a voltage source, rendering a voltage source inoperative, or deactivating the source, means making its voltage go to zero. In terms of a circuit diagram, zero voltage is accomplished by a ___________ ______________.

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Answer:

zero voltage is accomplished by a <u>short circuit</u>.

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Similarly, rendering a current source inoperative or deactivating it means making its current ____________________, which is accomplished in the circuit diagram by ______________________________.

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Answer:

making its current <u>go to zero</u>, which is accomplished in the circuit diagram by <u>open-circuiting the current source</u>.

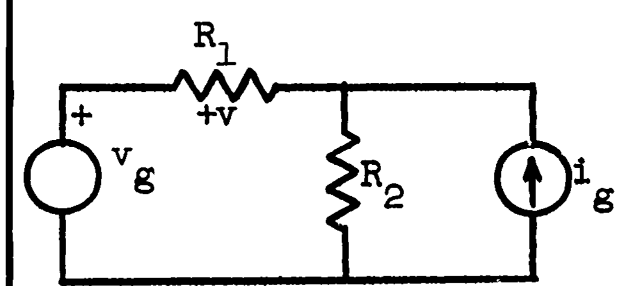

Figure 30

A procedure for solving network problems when more than one source (voltage or current) is present, then, is the following:

1. <u>Deactivate</u> all sources but one. This means short-circuit (ideal) voltage sources and open-circuit (ideal) current sources.
2. Use any method to find the desired voltages and currents due to this one source.
3. Repeat this process for each source in the network.
4. Add the results due to each source.

Let us illustrate the procedure in the simple network of Fig. 30. It is desired to find the voltage v across R_1. Let v_1 be the value of v when the only source is v_g, the current source being <u>deactivated</u>, and v_2 be the value of v when only i_g is present.

Redraw the diagram in these two cases and specify the disposition made of the sources.

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Answer:

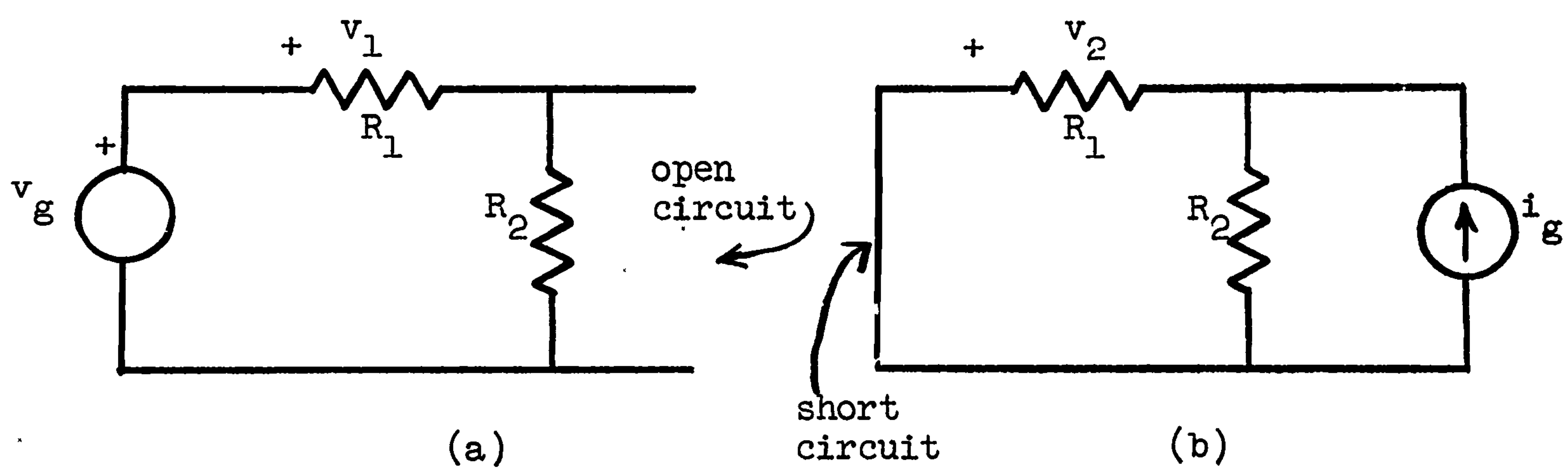

Figure 30

From these two diagrams compute v_1 and v_2 and write the desired voltage v. (Note the structure of Fig. 30(a)).

Answer:

$$v_1 = \frac{R_1}{R_1+R_2} v_g$$

$$v_2 = -\frac{R_1R_2}{R_1+R_2} i_g$$

$$v = v_1 + v_2 = \frac{R_1}{R_1+R_2} (v_g - R_2 i_g)$$

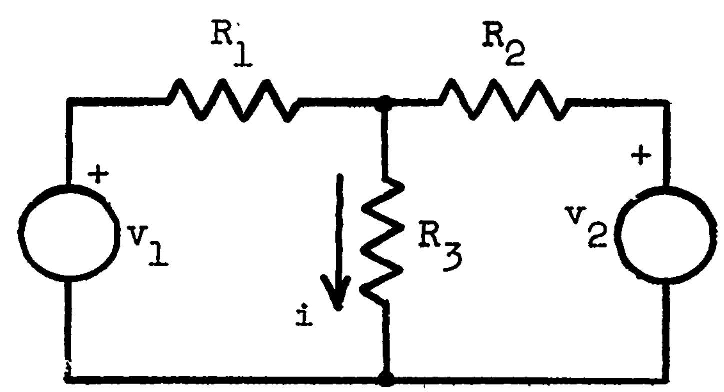

Figure 31

That was a relatively simple example and other methods of solution would be just as convenient to use.

A more complicated network is shown in Fig. 31.

It is desired to find the current i in resistor R_3. Let i_1 be the value of i when only source v_1 is operative, the other source being deactivated, and let i_2 be the value of i when only v_2 is operative.

Redraw the diagram in these two cases and show what is done with the sources.

Answer: To find i_1: To find i_2:

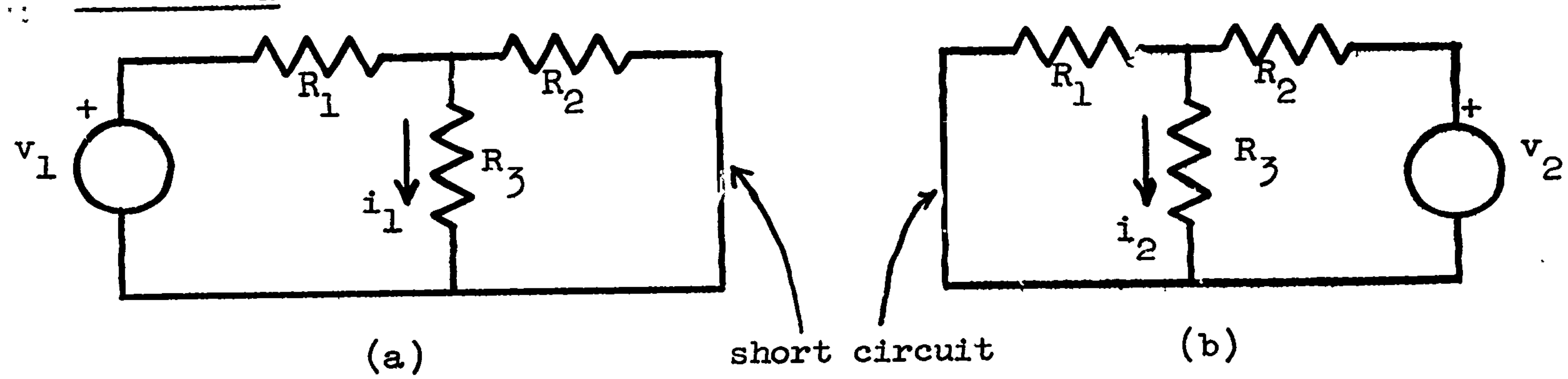

Figure 31

Let the resistors have the following numerical values: $R_1 = 30$, $R_2 = 5$ and $R_3 = 20$, all in ohms. In Fig. 31(a), R_2 and R_3 are in parallel. The current i_1 can be determined if the voltage across R_3 can be found. The structure of the network is that of a ________ ________. Hence, the voltage across R_3 will be __________ volts, and so i_1 = ____________ amps.

Answer:

The structure is a voltage divider.

The Voltage across R_3 will be $\dfrac{\dfrac{R_2R_3}{R_2+R_3}\,v_1}{\dfrac{R_2R_3}{R_2+R_3}+R_1} = \dfrac{2}{17}\,v_1$ volts

$i_1 = \dfrac{v_1}{170}$ amps.

The structure of Fig. 31(b) is similar to that of Fig. 31(a), but now R_1 and R_3 are in parallel. By a similar procedure, the current i_2 is found to be:

i_2 = ______________________.

By the principle of superposition, then, the total current i in resistor R_3 is:

i = ______________________.

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Answer:

$$i_2 = \frac{3}{85} v_2$$

$$i = \frac{v_1}{170} + \frac{3v_2}{85}$$

To achieve some insight into the relative ease or difficulty of using the principle of superposition, you should solve the same problem by the use of, say, loop equations.

<u>Summary</u>:

1. One procedure for solving network problems consists of (1) choosing a number of branch currents in terms of which all branch currents are expressed by applying Kcl; these currents are called the ____________; (2) next, Kvl equations are written around the closed paths, while simultaneously substituting for the voltages by Ohm's law. The resulting <u>loop equations</u> are a set of <u>linear</u> equations in as many unknowns as there are loop currents. After these equations are solved all other branch variables can be computed.

2. An alternative procedure is to write a set of <u>node equations</u>. The first step is to select a __________ node and to label all other node voltages. This is equivalent to the application of __________ ________ ________. The next step is ____

The resulting <u>node equations</u> are a set of <u>linear</u> equations in ________ (how many?) variables. After these are solved for the node voltages, all other branch variables can be determined.

<u>Summary (continued)</u>:

3. Because the <u>loop</u> and <u>node</u> equations are a set of <u>linear</u> equations in ____________, the principle of superposition applies. According to this principle, the current or voltage resulting from the simultaneous presence of a number of sources is ____

(If you need confirmation for any of your responses, look back over the material of the last section.)

This terminates the booklet on Resistive Networks.